BANA
UEBC SAMPLER 2

Prepared under the auspices of the
Braille Authority of North America

June 2001

Braille
Authority of
North
America

Contents

Introduction

The Braille Authority of North America (BANA) is pleased to send you UEBC Sampler 2. It illustrates the draft braille code being developed by the Unified English Braille Code Research Project under the direction of the International Council on English Braille (ICEB). If you have not seen UEBC Sampler 1, please contact the appropriate person from the list of addresses at the end of this Introduction.

UEBC Sampler 2 includes examples of published technical materials. Four of them, each 10 consecutive print pages, are brailled in UEBC and in the BANA code in use in North America for that subject, e.g. Nemeth Code for the algebra and calculus, Computer Code for the computer notation, and Chemistry Code for the chemistry. There are also: some elementary arithmetic samples; a list of examples used in her teaching by Susan Osterhaus, a mathematics teacher at the Texas School for the Blind and Visually Impaired; and an example provided by Jane Corcoran, a California transcriber.

At the beginning of each sample there is a list of the new and changed symbols you will encounter in UEBC. Each symbol is preceded by the dot locator (dots 46, 123456).

To help those who may not be familiar with the three present BANA technical codes, a list of symbols required to read the materials accompanies each sample.

The samples are organized so that the UEBC version is presented on the right-hand page with the corresponding material in the appropriate BANA technical code presented on the left-hand facing page.

UEBC format has not been finalized. These examples follow the same format used by the present BANA codes with one exception. UEBC follows the spacing of the original text. For instance, mathematical signs of operation are spaced or unspaced as they appear in print.

In the narrative portion of these examples you will notice that eight contractions found in English Braille American Edition (EBAE) are not used in UEBC. Six are brailled letter-for-letter: **ble**, **com**, **dd**, **ally**, **to**, **by**; and two are brailled with a contraction: **into** and **ation**. All of the other 181 contractions, wordsigns and shortforms are unchanged.

Each UEBC symbol is unambiguous—a print symbol is represented by the same braille symbol regardless of the subject. Because there are only 63 possible single cell dot combinations many symbols have to be made up of two or three cells. If the braille symbol uses the same dot configuration as a contraction it must be preceded by a grade one indicator to remove any ambiguity. This design feature will make it easier for a braille reader to work independently using computer translation and be assured of the accuracy of both the print and braille.

Nemeth Reading Notes

Nemeth numbers are brailled in the lower part of the cell. The number indicator is omitted when numbers immediately follow other symbols. Number indicators are also omitted when numbers are aligned one above the other, as in addition problems. A punctuation indicator is used where necessary to distinguish signs of punctuation from digits.

UEBC Reading Notes

UEBC uses grade one indicators to set off sections of text containing symbols that would otherwise be read as literary contractions. The number indicator also sets grade one mode for unspaced symbols or letters that follow a number.

Print Copies

To obtain a print copy of the examples in Sampler 2 or a print copy of Sampler 1 contact

The American Foundation for the Blind
National Literacy Center
Contact: Frances Mary D'Andrea
404-525-2303 or AFB's Information Center 800-232-5463
E-mail: literacy@afb.net

To obtain a braille copy of Sampler 1 contact

Kim Charlson, Braille Authority of North America,
617-972-7249 or e-mail charlsonk@perkins.pvt.k12.ma.us

For Canadians, print and braille copies of Sampler 1 may be obtained by contacting

Darleen Bogart
The Canadian National Institute for the Blind
416-480-7530 or 800-268-8818
E-mail: bogartd@lib.cnib.ca

Questionnaire

Thank you for reading UEBC Sampler 2. BANA wants your comments and has prepared a short list of questions as a guide (to get you started). You may respond in braille, print, on audio tape or by e-mail. Up-to-date information on UEBC is available by visiting the BANA web site at http://www.brailleauthority.org

Directions: Please complete the following questionnaire after reading through Sampler 2. Write your answers below, on a separate sheet of paper, or in an e-mail message. Send your completed survey to:

Warren Figueiredo
Louisiana Instructional Materials Center for the Blind
1230 Government Street
Baton Rouge, LA 70802
E-mail responses: UEBCinput@aol.com

Your input will be useful to the BANA Board in studying the UEBC. Thank you!

Questions about UEBC and Other Issues

1. Before you read Sampler 2, what were your feelings toward unifying the braille codes?

2. Having read through Sampler 2, how have your ideas/feelings changed toward a unified braille code?

3. What do you really like about the Unified English Braille Code (UEBC)?

4. What do you really dislike about the UEBC?

5. What issues do you feel the BANA Board should consider in making a decision on adoption of the UEBC?

6. Are your comments based on all six samples? Yes No
   If not, please check those which are included. 1 2 3 4 5 6

Optional Background Information

7. What kind of materials do you typically read in braille? Check all that apply.
   a. magazines
   b. fiction
   c. nonfiction
   d. textbooks
   e. other (list):

8. What math or technical materials do you read? Check all that apply.
   a. textbooks
   b. technical materials for my job
   c. computer braille
   d. chemistry or other scientific notation
   e. other (list):

9. What math code did you learn in school?
   a. upper numbers (Taylor Code)
   b. lower numbers (Nemeth)
   c. upper numbers (BAUK—Braille Authority of the United Kingdom)
   d. no math code used

10. When did you learn braille?
    a. before age 18
    b. 18-55 years of age
    c. 56 and above

Optional Personal Information

Name:
Age:
Gender:
Occupation:
Preferred Literacy Medium:
E-mail address:
Address:
Telephone:

Sample 1.
Spatially Arranged Mathematics

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).

1a. Arithmetic Problems

Add.

28. $\begin{array}{r} 8 \\ \underline{+5} \end{array}$ **29.** $\begin{array}{r} 5 \\ \underline{+8} \end{array}$ **30.** $\begin{array}{r} 2 \\ \underline{+7} \end{array}$ **31.** $\begin{array}{r} 7 \\ \underline{+2} \end{array}$ **32.** $\begin{array}{r} 3 \\ \underline{+0} \end{array}$ **33.** $\begin{array}{r} 8 \\ \underline{+0} \end{array}$

34. $(2 + 6) + 5$ **35.** $2 + (6 + 5)$ **36.** $(7 + 3) + 0$ **37.** $4 + 5 + 8$

Find the sums and differences. Use a calculator if necessary.

1. $\begin{array}{r} 2{,}964 \\ \underline{+\ 5{,}682} \end{array}$ **2.** $\begin{array}{r} 6{,}587 \\ \underline{+\ 2{,}744} \end{array}$ **3.** $\begin{array}{r} 4{,}532 \\ \underline{+\ 1{,}607} \end{array}$ **4.** $\begin{array}{r} 4{,}430 \\ \underline{-\quad 726} \end{array}$ **5.** $\begin{array}{r} 6{,}429 \\ \underline{-\ 5{,}161} \end{array}$ **6.** $\begin{array}{r} 7{,}000 \\ \underline{-\ 2{,}674} \end{array}$

Find the products.

1. $\begin{array}{r} 4 \\ \underline{\times 1} \end{array}$ **2.** $\begin{array}{r} 1 \\ \underline{\times 3} \end{array}$ **3.** $\begin{array}{r} 2 \\ \underline{\times 0} \end{array}$ **4.** $\begin{array}{r} 0 \\ \underline{\times 3} \end{array}$ **5.** $\begin{array}{r} 2 \\ \underline{\times 1} \end{array}$ **6.** $5 \times 4 = 20$
$4 \times 5 = n$

Arithmetic Problems in Nemeth Code

Nemeth Symbols

⠸	punctuation indicator
⠬	plus sign
⠤	minus sign
⠈⠡	times sign (cross)
⠨⠅	equal sign
⠷	opening parenthesis
⠾	closing parenthesis
⠠	mathematical comma

Arithmetic Problems in UEBC

UEBC Symbols

⠐⠖	plus sign
⠐⠤	minus sign
⠐⠦	times sign (cross)
⠐⠶	equal sign
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis

1b. System of Equations

Equating coefficients leads to the system

$$
\begin{aligned}
c_1 + {} & 2c_2 + {} & c_3 &= 0 \\
-2c_1 + {} & c_2 + {} & 8c_3 &= 0 \\
3c_1 + {} & 8c_2 + {} & 7c_3 &= 0
\end{aligned}
$$

System of Equations in Nemeth Code

Nemeth Symbols

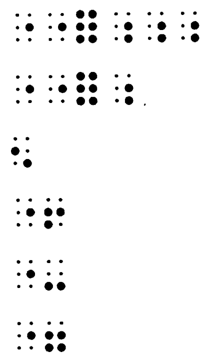
⠬	plus sign
⠤	minus sign
⠨⠅	equal sign

⠠⠑⠟⠥⠁⠞⠬ ⠉⠕⠑⠖⠊⠉⠊⠢⠞⠎ ⠇⠂⠙⠎ ⠖⠮ ⠎⠽⠌⠑⠍

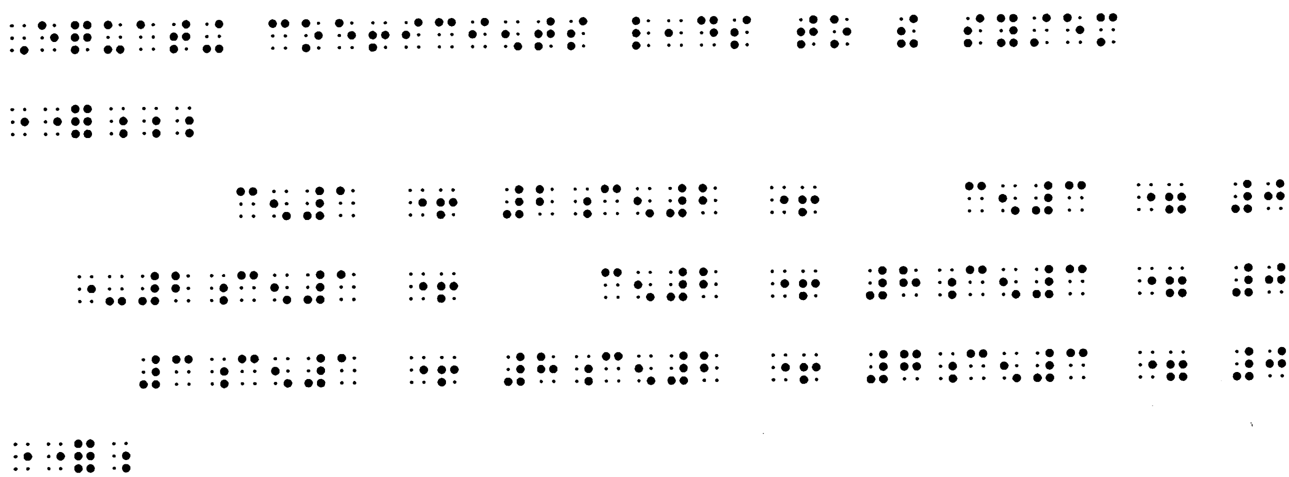
⠉⠂⠬⠆⠉⠆⠬ ⠉⠒ ⠨⠅ ⠼⠴
⠤⠼⠆⠉⠂⠬ ⠉⠆⠬⠦⠉⠒ ⠨⠅ ⠼⠴
⠼⠒⠉⠂⠬⠦⠉⠆⠬⠶⠉⠒ ⠨⠅ ⠼⠴

System of Equations in UEBC

UEBC Symbols



	begin grade one passage
	end grade one passage
	subscript next item
	plus sign
	minus sign
	equal sign



1c. Matrix Multiplication

For each fixed x in $[a, b]$, the matrix equation

$$\begin{pmatrix} f_1(x) & f_2(x) & \text{ê} & f_n(x) \\ f_1'(x) & f_2'(x) & \text{ê} & f_n'(x) \\ \text{Ë} & & & \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & \text{ê} & f_n^{(n-1)}(x) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \text{Ë} \\ \alpha_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \text{Ë} \\ 0 \end{pmatrix} \quad (11)$$

Matrix Multiplication in Nemeth Code

Nemeth Symbols

⠷	opening parenthesis
⠾	closing parenthesis
⠠⠷	multiline opening parenthesis
⠠⠾	multiline closing parenthesis
⠈⠷	opening bracket
⠈⠾	closing bracket
⠠	mathematical comma
⠰	subscript indicator
⠘	superscript indicator
⠐	return to baseline after subscript or superscript
⠄	prime sign
⠤	minus sign
⠨⠅	equal sign
⠨⠁	Greek alpha

Matrix Multiplication in UEBC

UEBC Symbols

⠐⠐⠿⠰⠰⠰	begin grade one passage
⠐⠐⠿⠰	end grade one passage
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠨⠣	opening bracket
⠨⠜	closing bracket
⠠⠐⠣	multiline opening parenthesis
⠠⠐⠜	multiline closing parenthesis
⠢	subscript next item
⠔	superscript next item
⠲⠲⠲	ellipsis
⠲⠻⠲⠻⠲	vertical ellipsis
⠶	prime sign
⠣	begin compound item
⠜	end compound item
⠐⠤	minus sign
⠐⠶	equal sign
⠨⠁	Greek alpha

1d. Long Division

4. Bring down the next digit in the dividend. Repeat these steps until there are no digits left to bring down.

```
      620 R19
  32)19,859
    -19 2
       65
      -64
        19
        -0
        19
```

Remember to check the answer.

Long Division in Nemeth Code

Nemeth Symbols

⠸	punctuation indicator
⠐	baseline indicator
⠕	curved division sign
⠠	mathematical comma
⠤	minus sign

Long Division in UEBC

UEBC Symbols

⠐⠜	curved division sign
⠐⠤	minus sign
⠐	numeric space

1e. Long Multiplication

2. Multiply by the tens digit.

$$\begin{array}{r} {\scriptstyle 1} \\ 643 \\ \underline{\times 35} \\ 3215 \\ 19290 \end{array} \leftarrow 30 \times 643$$

↑

You do not have to write the zero. Start the answer in the tens place.

Remember to regroup.

Long Multiplication in Nemeth Code

Nemeth Symbols	
⠸	punctuation indicator
⠶⠶⠶	carry line
⠈⠡	times sign (cross)

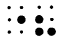
⠼⠆⠸⠲ ⠠⠍⠥⠇⠞⠊⠏⠇⠽ ⠴⠮ ⠞⠢⠎ ⠙⠊⠛⠊⠞⠲

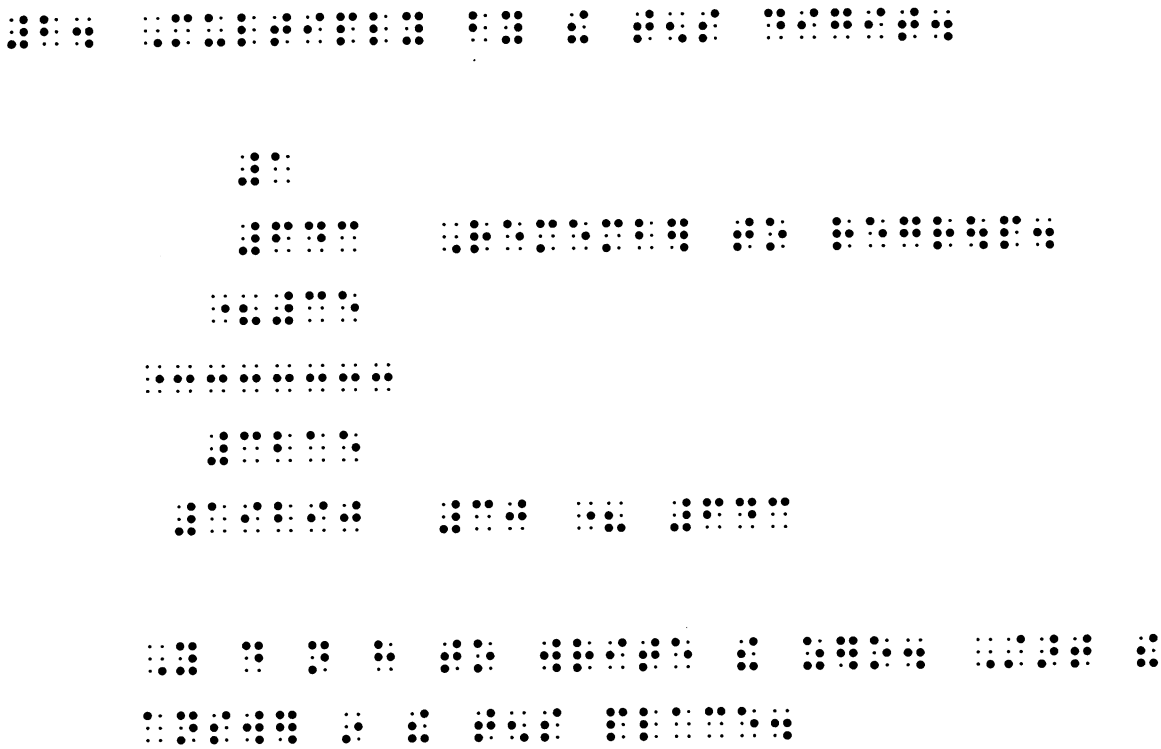
```
   ⠂
⠶⠶⠶⠶⠶⠶⠶
   ⠖⠲⠒  ⠠⠗⠑⠍⠑⠍⠃⠻ ⠖⠗⠑⠛⠗⠳⠏⠲
  ⠈⠡⠒⠢
⠒⠒⠒⠒⠒⠒⠒
  ⠒⠆⠂⠢
 ⠂⠔⠆⠔⠴  ⠼⠒⠴⠈⠡⠖⠲⠒
```

⠠⠽ ⠙ ⠝ ⠓ ⠖⠺⠗⠊⠞⠑ ⠮ ⠵⠻⠕⠲ ⠠⠌⠜⠞ ⠮ ⠁⠝⠎⠺⠻
⠔ ⠮ ⠞⠢⠎ ⠏⠇⠁⠉⠑⠲

Long Multiplication in UEBC

UEBC Symbols

 times sign (cross)



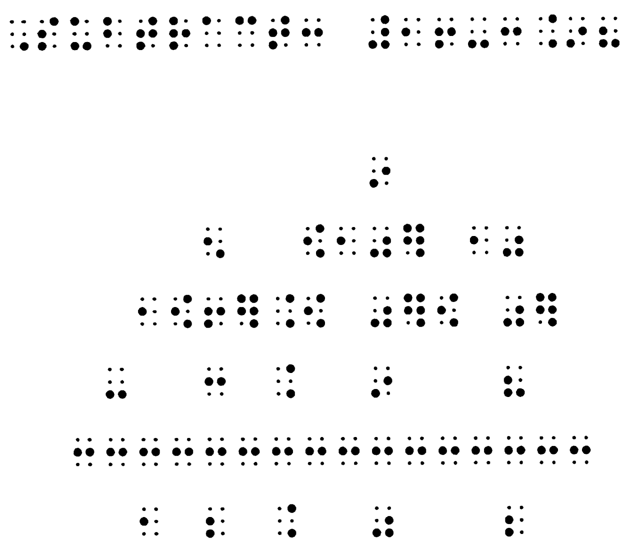
1f. Cancelled Digits

Subtract: 16 - 3.98

```
     9
  5 1̸0̸10
 1 6̸.0̸0̸
 - 3.9 8
 -------
 1 2.0 2
```

Cancelled Digits in Nemeth Code

Nemeth Symbols	
⠪	opening cancellation indicator
⠻	closing cancellation indicator
⠤	minus sign
⠨	decimal point



Cancelled Digits in UEBC

UEBC Symbols	
⠐⠤	minus sign
⠈⠱	line through previous item
⠲	decimal point

⠠⠎⠥⠃⠞⠗⠁⠉⠞⠒ ⠼⠁⠋ ⠐⠤ ⠼⠉⠲⠊⠓

```
             ⠼⠊
       ⠼⠑   ⠼⠁⠚⠈⠱⠼⠁⠚
     ⠼⠁⠼⠋⠈⠱⠲ ⠼⠚⠈⠱ ⠼⠚⠈⠱
   ⠐⠤  ⠼⠉  ⠲ ⠼⠊   ⠼⠓
  ⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
     ⠼⠁⠼⠃  ⠲ ⠼⠚   ⠼⠃
```

1g. Cancelled Fractions

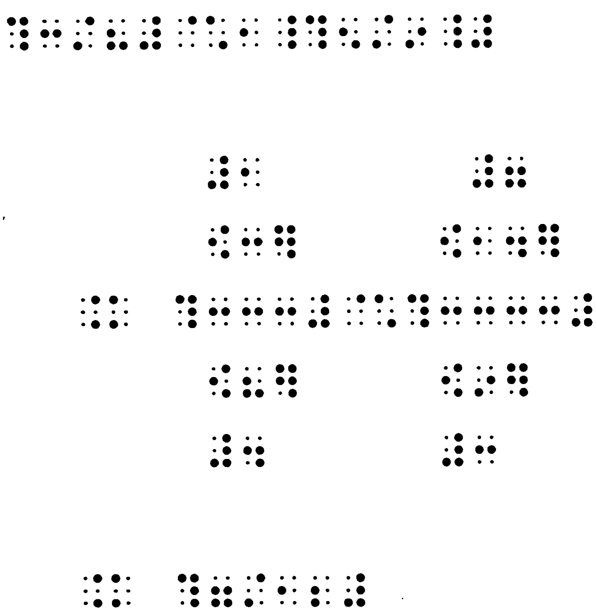
$$\frac{3}{8} \times 1\frac{5}{9} = \frac{\overset{1}{\cancel{3}}}{\underset{4}{\cancel{8}}} \times \frac{\overset{7}{\cancel{14}}}{\underset{3}{\cancel{9}}}$$

$$= \frac{7}{12}$$

Cancelled Fractions in Nemeth Code

Nemeth Symbols

⠪	opening cancellation indicator
⠻	closing cancellation indicator
⠈⠡	times sign (cross)
⠨⠅	equal sign
⠹	opening fraction indicator
⠼	closing fraction indicator
⠸⠹	opening mixed fraction indicator
⠸⠼	closing mixed fraction indicator
⠌	horizontal fraction line



Cancelled Fractions in UEBC

UEBC Symbols	
⠐⠦	times sign (cross)
⠐⠶	equal sign
⠈⠱	line through previous item

⠼⠉⠌⠓ ⠐⠦ ⠼⠁⠼⠑⠌⠊

```
          ⠼⠁              ⠼⠛
          ⠼⠉⠈⠱            ⠼⠁⠙⠈⠱
   ⠐⠶   ⠐⠒⠒⠒⠒⠒   ⠐⠦   ⠐⠒⠒⠒⠒⠒⠒
          ⠼⠓⠈⠱            ⠼⠊⠈⠱
          ⠼⠙              ⠼⠉
```

⠐⠶ ⠼⠛⠌⠁⠃

1h. Number Line

Use fractions, mixed numbers, and decimals to name points on the number line. Write each missing number.

1.

2	$2\frac{1}{10}$	$2\frac{2}{10}$	$2\frac{3}{10}$	___	$2\frac{5}{10}$	$2\frac{6}{10}$	___	$2\frac{8}{10}$	$2\frac{9}{10}$	3	$3\frac{1}{10}$	$3\frac{2}{10}$
2.0	2.1	___	2.3	2.4	2.5	2.6	___	2.8	2.9	3.0	___	3.2

Number Line in Nemeth Code

Nemeth Symbols	
⠸	punctuation indicator
⠪	left arrowhead
⠒⠒⠒	horizontal shaft of number line
⠳	vertical mark
⠕	right arrowhead
⠸⠹	opening mixed fraction indicator
⠸⠼	closing mixed fraction indicator
⠌	horizontal fraction line
⠿	missing number (blank underline)

Number Line in UEBC

UEBC Symbols

⠰	grade one symbol
⠳⠪	left arrowhead
⠒⠒⠒	horizontal shaft of number line
⠺	vertical mark
⠐	(at end of line) continuation indicator
⠳⠕	right arrowhead
⠠⠤	dash (used for blank underline)
⠲	decimal point

1i. Addition Puzzle

Each letter in these problems represents a different digit.

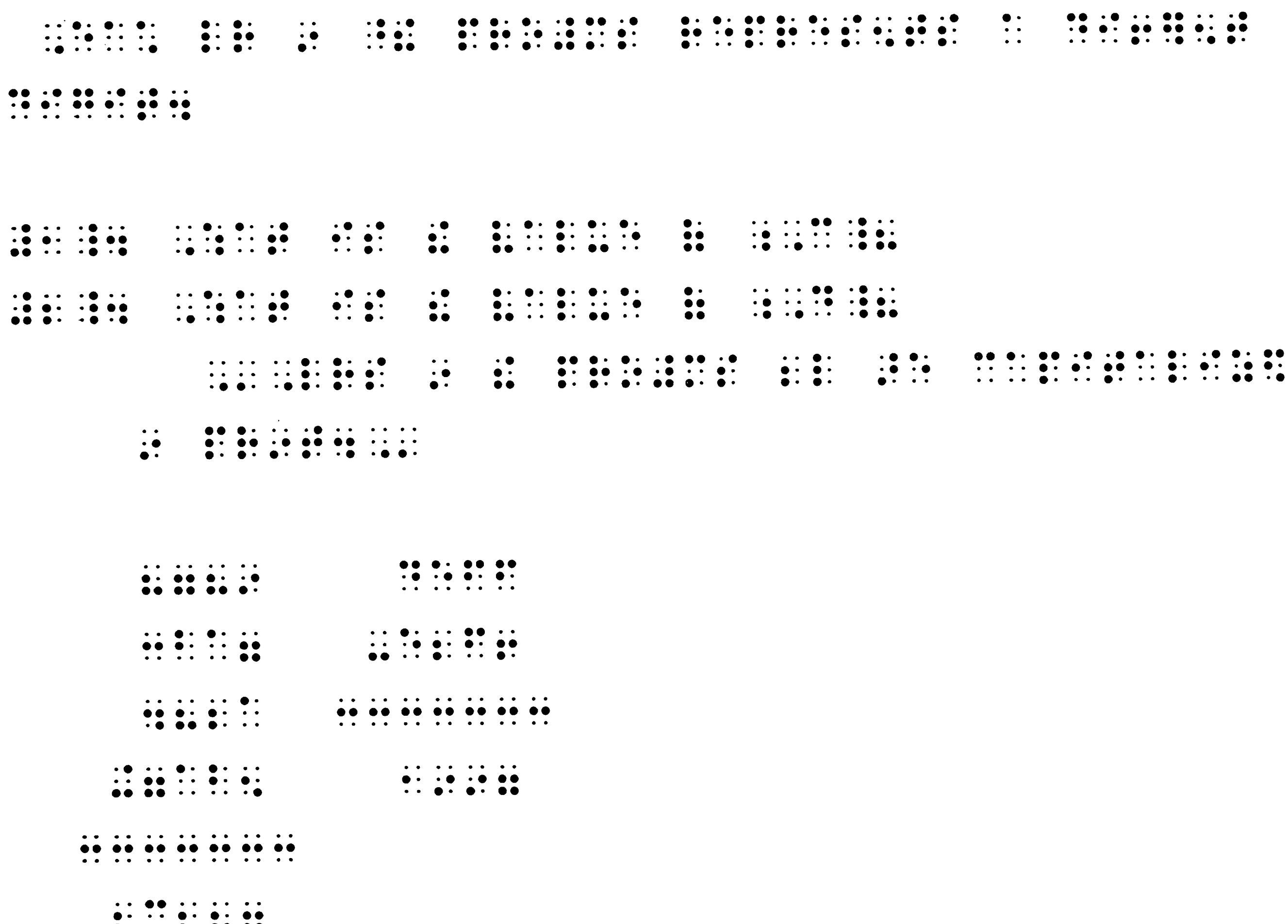
1. What is the value of C?
2. What is the value of D?

```
 8789          DEFF
 3BA7         -E2F6
 482A         -----
+7AB5          1997
-----
2C287
```

Addition Puzzle in Nemeth Code

Nemeth Symbols

⠸	punctuation indicator
⠠⠄	transcriber's note indicator
⠬	plus sign
⠤	minus sign



Addition Puzzle in UEBC

UEBC Symbols

⠐⠖	plus sign
⠐⠤	minus sign

⠠⠑⠁⠡ ⠇⠗ ⠔ ⠘⠮ ⠏⠗⠕⠃⠇⠑⠍⠎ ⠗⠑⠏⠗⠑⠎⠢⠞⠎ ⠁ ⠙⠊⠖⠻⠢⠞ ⠙⠊⠛⠊⠞⠲

⠼⠁⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠠⠉⠦
⠼⠃⠲ ⠠⠱⠁⠞ ⠊⠎ ⠮ ⠧⠁⠇⠥⠑ ⠷ ⠰⠠⠙⠦

```
   ⠼⠓⠼⠛⠼⠓⠼⠊     ⠠⠙⠠⠑⠠⠋⠠⠋
   ⠼⠉⠠⠃⠠⠁⠼⠛   ⠐⠤⠠⠑⠼⠃⠠⠋⠼⠋
   ⠼⠙⠼⠓⠼⠃⠠⠁  ⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 ⠐⠖⠼⠛⠠⠁⠠⠃⠼⠑     ⠼⠁⠼⠊⠼⠊⠼⠛
⠐⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒⠒
 ⠼⠃⠠⠉⠼⠃⠼⠓⠼⠛
```

Sample 2. Algebra

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).

Use synthetic division to find the quotient and remainder in the following divisions and express the answer in the form of Eq. (3-9).

9. $x^3 - 2x^2 + 3x - 4$ by $x - 3$
10. $2x^3 + x^2 - x + 4$ by $x + 1$
11. $x^4 - 5x^3 + x^2 - 6$ by $x - 1$
12. $x^3 + 3x^2 - 2x - 5$ by $x + 2$
13. Find the value of $x^3 - 2x^2 + 3x - 4$ at $x = 3$.
14. Find the value of $2x^3 + x^2 - x + 4$ at $x = -1$.
15. Find the value of $x^4 - 5x^3 + x^2 - 6$ at $x = 1$.
16. Find the value of $x^3 + 3x^2 - 2x - 5$ at $x = -2$.
17. Compare the values obtained in Problems 13 through 16 with the remainders in Problems 9 through 12.
18. a) Evaluate $ax^2 + bx + c$ at $x = p$.
    b) Divide $ax^2 + bx + c$ by $x - p$.
    c) Compare the remainder found in (b) with the value obtained in (a).
    d) Would you expect the same type of result for any degree polynomial?

▶ 3-4 SPECIAL PRODUCTS

There are certain special products which occur so frequently in algebra that they have been classified. These are given below. *The letters in the formulas may stand for any algebraic expression.* Each is a direct result of the axioms in Chapter 2. The reader should not only verify each by actually carrying out the steps and giving the reasons, but also memorize them, so that he can recognize both the product from the factors and the factors from the product.

$$a(x + y) \equiv ax + ay. \tag{3-11}$$

$$(x + y)(x - y) \equiv x^2 - y^2. \tag{3-12}$$

$$(x \pm y)^2 \equiv x^2 \pm 2xy + y^2. \qquad (3\text{-}13)^*$$

$$(x + a)(x + b) \equiv x^2 + (a + b)x + ab. \tag{3-14}$$

$$(ax + b)(cx + d) \equiv acx^2 + (ad + bc)x + bd. \tag{3-15}$$

$$(x \pm y)^3 \equiv x^3 \pm 3x^2y + 3xy^2 \pm y^3. \tag{3-16}$$

$$(x \pm y)(x^2 \mp xy + y^2) \equiv x^3 \pm y^3. \tag{3-17}$$

The reader should determine which of the above formulas is used in the following illustrations.

---

* The sign $\pm$ is read "plus or minus." If the upper (lower) sign is used in the left member, it is also used in the right, so that $(x \pm y)^2 \equiv x^2 \pm 2xy + y^2$ means $(x + y)^2 \equiv x^2 + 2xy + y^2$ and $(x - y)^2 \equiv x^2 - 2xy + y^2$.

Illustration 1. $(2x^2 - 3y)(2x^2 + 3y) \equiv (2x^2)^2 - (3y)^2$
$\equiv 4x^4 - 9y^2.$

Illustration 2. $(x + 2)(x + 5) \equiv x^2 + (2 + 5)x + 10$
$\equiv x^2 + 7x + 10.$

Illustration 3. $(3x + 4y)(2x - 3y) \equiv 6x^2 + (-9 + 8)xy - 12y^2$
$\equiv 6x^2 - xy - 12y^2.$

Illustration 4

$$\begin{aligned}(x + y - 1)^3 &\equiv [(x + y) - 1]^3\\ &\equiv (x + y)^3 - 3(x + y)^2 + 3(x + y) - 1\\ &\equiv x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2 - 6xy - 3y^2 + 3x + 3y - 1.\end{aligned}$$

Here $(x + y)$ is considered first as one term.

Illustration 5

$$\begin{aligned}(3x + 2y)(9x^2 - 6xy + 4y^2) &\equiv (3x + 2y)[(3x)^2 - (3x)(2y) + (2y)^2]\\ &\equiv (3x)^3 + (2y)^3\\ &\equiv 27x^3 + 8y^3.\end{aligned}$$

PROBLEMS

Find the following products.

1. $2a(3x - 4y)$
2. $-3x(2x + 7y)$
3. $-7xy(3x^2 + 4y)$
4. $4x^2yz(z^2 + xy + yz)$
5. $(2x - 3y)(2x + 3y)$
6. $(7x + 5y^2)(7x - 5y^2)$
7. $(x + 2y)(x - 2y)(x^2 + 4y^2)$
8. $(x - 3)^2$
9. $(2x + 7y)^2$
10. $(3x^2y - 5z^2)^2$
11. $(x - 2)(x - 5)$
12. $(2x + 3)(x - 5)$
13. $(xy^2 - z^2w)^2$
14. $(\frac{1}{2}x + \frac{2}{3}y)^2$
15. $(4x - 3y)(7x + 3y)$
16. $[(x + 1) - z][(x + 1) + z]$
17. $(2x + 3y + 3)(2x + 3y - 3)$
18. $(2x + 3y + 4z)^2$
19. $(x - 2y - z)^2$
20. $(2a + b)^3$
21. $(x + 2)(x^2 - 2x + 4)$
22. $(x - 3)(x^2 + 3x + 9)$
23. $(x + 3y + 2z - 4w)(x + 3y - 2z + 4w)$
24. $(4x - 2y - 3z + 3w)(4x + 2y + 3z + 3w)$
25. $(a - b + c - d)^2$
26. $(2a + 3b - c - 4d)^2$
27. $[2(x + 2y) - 3][2(x + 2y) + 4]$
28. $[2(x - 3y) + 5][3(x - 3y) - 2]$
29. $(2x + 3y)^3$
30. $(5x - 3y)^3$

3-5 FACTORS AND FACTORING

The process of factoring an algebraic expression is similar to that of finding the factors of a composite number. Recall the discussion of prime and composite integers in Article 1-4. This process, which is usually restricted at this elementary stage to factoring polynomials with rational coefficients and to factors completely free from irrational numbers, is frequently performed by reversing the processes considered in Article 3-4. Such a factorization is considered complete when each algebraic factor is a *prime factor;* that is, an algebraic expression that cannot be factored without violating the above restrictions.

The more common types of factoring are illustrated below. Note the importance and application of the distributive axioms in this discussion.

Example 1. Factor $2ax^2 - 4ay^2 + 8a^2x$.

Solution. The polynomial in this problem has $2a$ as a common factor.

$$2ax^2 - 4ay^2 + 8a^2x \equiv 2a(x^2 - 2y^2 + 4ax).$$

Example 2. Factor $x(a + 2b) - 3y(a + 2b)$.

Solution. Each of the two expressions has the common term $(a + 2b)$. Therefore,

$$x(a + 2b) - 3y(a + 2b) \equiv (x - 3y)(a + 2b).$$

Example 3. Factor $(4x^2/y^2) - (9a - b)^2$.

Solution. This expression is the difference between two perfect squares.

$$\begin{aligned}\frac{4x^2}{y^2} - (9a - b)^2 &\equiv \left(\frac{2x}{y}\right)^2 - (9a - b)^2 \\ &\equiv \left[\frac{2x}{y} + (9a - b)\right]\left[\frac{2x}{y} - (9a - b)\right] \\ &\equiv \left(\frac{2x}{y} + 9a - b\right)\left(\frac{2x}{y} - 9a + b\right).\end{aligned}$$

Example 4. Factor $9x^2 - 30xy + 25y^2$.

Solution. This algebraic expression is a perfect square.

$$9x^2 - 30xy + 25y^2 \equiv (3x - 5y)^2.$$

Example 5. Factor $27x^3 + (8/y^3)$.

Solution. The algebraic expression is the sum of two cubes. Accordingly,

$$27x^3 + \frac{8}{y^3} \equiv \left(3x + \frac{2}{y}\right)\left(9x^2 - \frac{6x}{y} + \frac{4}{y^2}\right).$$

Example 6. Factor $12x^2 + 7xy - 10y^2$.

Solution. This trinomial in the form of Eq. (3–15) is factored by trial and error. The result will be in the form $(ax + by)(cx + dy)$, where $ac = 12$, $bd = -10$, and $ad + bc = 7$. Here a and c are both plus, and b and d are different in sign. The correct combination, we find, is $12x^2 + 7xy - 10y^2 \equiv (4x + 5y)(3x - 2y)$.

Example 7. Factor $6x^4 + 7x^2y^2 - 3y^4$.

Solution. This is the same type as Example 6.

$$6x^4 + 7x^2y^2 - 3y^4 \equiv (3x^2 - y^2)(2x^2 + 3y^2).$$

Although the first factor on the right is the difference of two squares, it cannot be factored further, for such factorization would introduce irrational quantities.

PROBLEMS

Factor the following completely.

1. $4x - 20$
2. $10x + 15yz$
3. $3y^2 - 9y$
4. $4x^3y^2 + 6x^2y^3$
5. $xy^2z^3 - 3x^2yz^2 + 5xy^3z^2$
6. $a^2b^3c^4 - a^3b^4c^5 + 2a^2b^4c^4$
7. $3y(2x + 5) - 4x(2x + 5)$
8. $3y(4 - y) + 6x^2(4 - y)$
9. $2z^2(x + 3y) - 6xz(x + 3y)$
10. $3x(3 - 2y) - 2xy(3 - 2y)$
11. $9 - a^2$
12. $16x^2 - 9y^2$
13. $225a^8 - 64b^2$
14. $(c^6/d^8) - 121$
15. $x^3y^4 - 25xd^6$
16. $0.01x^4 - 196y^8$
17. $(x + 2y)^2 - z^2$
18. $(3x - 2y)^2 - 25z^2$
19. $(a + b)^2 - (c + d)^2$
20. $9(2x - y)^2 - 4(2a + b)^2$
21. $81(4x - 3y)^2 - 25(3z + w)^2$
22. $x^2 + 6x + 9 - (y^2 + 4y + 4)$
23. $x^2 - 8x + 16$
24. $4a^2 - 12ab + 9b^2$
25. $66xy + 9x^2y^2 + 121$
26. $2x^3 - 28x^2 + 98x$
27. $5z^2 - 30wz + 45w^2$
28. $x^{2n} + 2x^ny^n + y^{2n}$
29. $(3 - x)^2 + 8(3 - x) + 16$
30. $25 - 30(2x - 3y) + 9(2x - 3y)^2$
31. $a^3 - 8$
32. $1 + (8/x^9)$
33. $8x^{6n} + 27y^{3m}$
34. $x^3 - (y^3/64)$
35. $27(x - y)^3 - 8(x + y)^3$
36. $5(a - 2b)^3 - 625(a - 2b)^3$
37. $x^2 - 7x + 12$
38. $y^2 - 2y - 8$
39. $a^2b^2 - ab - 20$
40. $2x^2 + 8x + 6$
41. $35x^2 - 24x + 4$
42. $3y^2 - y - 10$
43. $6a^2 + 7a - 20$
44. $2x^2 - 23xy - 39y^2$

45. $(x + y)^2 - 7(x + y) + 10$
46. $(y + z)^2 + (y + z) - 42$
47. $2(2x + y)^2 - (2x + y) - 10$
48. $6(x + y)^2 + 5(x + y)(y + z) - 6(y + z)^2$
49. $12(a + b)^2 - 14(a + b)(c + d) - 10(c + d)^2$
50. $4(x - 2)^2 + 5(x - 2)(y + 4) - 21(y + 4)^2$

There are many algebraic expressions which, by proper grouping, can be put into one of the forms in the previous examples and then factored.

Example 8. Factor $ax - ay - bx + by$.

Solution. If, by the associative axiom, we group the first two terms together, and the last two together, and (use the distributive axiom) factor out the common term, we transform the expression into the form of Example 2.

$$\begin{aligned} ax - ay - bx + by &\equiv a(x - y) - b(x - y) \\ &\equiv (x - y)(a - b). \end{aligned}$$

Example 9. Factor $4x^3 - 12x^2 - x + 3$.

Solution. Again we group the first two terms and the last two terms.

$$\begin{aligned} 4x^3 - 12x^2 - x + 3 &\equiv 4x^2(x - 3) - (x - 3) \\ &\equiv (x - 3)(4x^2 - 1) \\ &\equiv (x - 3)(2x + 1)(2x - 1). \end{aligned}$$

In both these examples we could have grouped the first and third, and the second and fourth terms, and obtained the same result.

Example 10. Factor $4x^2 - 12xy + 9y^2 + 4x - 6y - 3$.

Solution. If we group the first three terms, the solution becomes clear.

$$\begin{aligned} 4x^2 - 12xy + 9y^2 + 4x - 6y - 3 &\equiv (2x - 3y)^2 + 2(2x - 3y) - 3 \\ &\equiv [(2x - 3y) + 3][(2x - 3y) - 1] \\ &\equiv (2x - 3y + 3)(2x - 3y - 1). \end{aligned}$$

Example 11. Factor $x^4 + 2x^2y^2 + 9y^4$.

Solution. If the coefficient of the second term were 6, the expression would be a perfect square. Therefore, if we add (and subtract) $4x^2y^2$, our solution becomes evident.

$$\begin{aligned} x^4 + 2x^2y^2 + 9y^4 &\equiv x^4 + 6x^2y^2 + 9y^4 - 4x^2y^2 \\ &\equiv (x^2 + 3y^2)^2 - (2xy)^2 \\ &\equiv (x^2 + 3y^2 + 2xy)(x^2 + 3y^2 - 2xy). \end{aligned}$$

PROBLEMS

Factor the following expressions.

1. $ax - ay - by + bx$
2. $ax - 2ay - 6by + 3bx$
3. $x^3 - 2x^2 + 4x - 8$
4. $y^3 - 2y^2 + 5y - 10$
5. $2a - 6 - ab^2 + 3b^2$
6. $x^3 + 3x^2 - 9x - 27$
7. $x^2 - 2x + 1 - y^2$
8. $xy^3 + 2y^2 - xy - 2$
9. $4x^2 - y^2 + 4y - 4$
10. $x^6 - 7x^3 - 8$
11. $x^2 + 2xy + y^2 - z^2 + 2zw - w^2$
12. $4a^2 - x^2 + b^2 - y^2 - 4ab - 2xy$
13. $x^2 + 4xy + 4y^2 - x - 2y - 6$
14. $x^3 - 5x^2 - x + 5$
15. $x^4 - 7x^2y^2 + 9y^4$
16. $y^4 + y^2 + 25$
17. $a^4 + 2a^2b^2 + 9b^4$
18. $x^4 + 5x^2 + 9$
19. $b^4 + 6b^2c^2 + 25c^2$
20. $25x^2 + 30xy + 9y^2 + 15x + 9y + 2$
21. $3ax - 6ay + 4bx - 8by + cx - 2cy$
22. $20xy + 7zw - 5yz - 28xw$
23. $z^4 + 4z^3 - 2z - 8$
24. $x^4 + 4y^4$
25. $a^8 - b^8$
26. $x^6 + 1$
27. $x^2 + 2xy - z^2 - 2yz$
28. $(x^2 + 2x - 3)^2 - 4$
29. $(x - y - 2z)^2 - (2x + y - z)^2$
30. $2(x + 2)^2(x - 3) + 3(x + 2)(x - 3)^2$

3-6 SIMPLIFICATION OF FRACTIONS

A basic principle for fractions, algebraic as well as arithmetic, states that the value of a fraction is not changed if its numerator and denominator are both multiplied or both divided by the same quantity (not zero). This principle was stated in Theorem 2-8. Hence, the simplification or reduction of a fraction to lowest terms is always possible. Factor both the numerator and denominator into their prime factors and, by using the basic principle, divide the numerator and denominator by the product of all their common factors.

Example 1. Reduce $(8x^4y^7)/(12x^6y^3)$ to lowest terms.

Solution

$$\frac{8x^4y^7}{12x^6y^3} = \frac{2^3x^4y^7}{2^2 \cdot 3x^6y^3} = \frac{2^2x^4y^3 \cdot 2y^4}{2^2x^4y^3 \cdot 3x^2}.$$

By dividing both numerator and denominator by $2^2x^4y^3$, we have

$$\frac{8x^4y^7}{12x^6y^3} = \frac{2y^4}{3x^2}.$$

Example 2. Reduce $(x^2 - 7x + 10)/(2x^2 - x - 6)$ to lowest terms.

Solution. If we factor both numerator and denominator, we have

$$\frac{x^2 - 7x + 10}{2x^2 - x - 6} \equiv \frac{(x - 5)(x - 2)}{(2x + 3)(x - 2)},$$

and dividing both numerator and denominator by $x - 2$, that is, applying Theorem 2-8, we get

$$\frac{x^2 - 7x + 10}{2x^2 - x - 6} \equiv \frac{x - 5}{2x + 3}.$$

The elimination of a common factor by dividing the numerator and denominator of a fraction by this factor is called *multiplicative cancellation.* Such a process should be done with care, for Theorem 2-8 is true only when $x \neq 0$. In this case the identity is true for all values of x except $x = 2$ or $x = -\frac{3}{2}$, which are not permissible values.

Example 3. Reduce $(12x^2 + 30x - 72)/(52x - 8x^2 - 60)$ to lowest terms.

Solution

$$\frac{12x^2 + 30x - 72}{52x - 8x^2 - 60} \equiv \frac{6(2x - 3)(x + 4)}{4(3 - 2x)(x - 5)} \equiv \frac{3(x + 4)}{2(5 - x)}.$$

This identity follows from the fact that $2x - 3 = -(3 - 2x)$. (Recall Problem 4, Article 2-4.)

PROBLEMS

Reduce the following to lowest terms.

1. $\dfrac{28}{63}$
2. $\dfrac{27x^3}{225x^5}$
3. $\dfrac{a^4x^3y}{a^2xy^3}$
4. $\dfrac{a^2 + ab}{3a + 2a^3}$
5. $\dfrac{a^2x - a^2y}{ax^2 - ay^2}$
6. $\dfrac{24a^2}{6a^2 - 9a}$
7. $\dfrac{x^2 - 1}{x^2 - x}$
8. $\dfrac{x^2 - 4x + 4}{x^2 - 4}$
9. $\dfrac{x^2 - 16}{x^2 - 8x + 16}$
10. $\dfrac{a^2 - 3a - 4}{a^2 - a - 12}$
11. $\dfrac{y^2 - y - 6}{y^2 + 2y - 15}$
12. $\dfrac{2x^2 + 5x - 12}{4x^2 - 4x - 3}$
13. $\dfrac{6a^2 - 7a - 3}{4a^2 - 8a + 3}$
14. $\dfrac{ax + ay - bx - by}{am - bm - an + bn}$

15. $\dfrac{14x - 24 - 2x^2}{x^2 + x - 20}$

16. $\dfrac{(4x^2 - 9y^2)(18x - 12)}{(2x - 3y)(12x - 8)}$

17. $\dfrac{x^2 - 36}{x^3 - 216}$

18. $\dfrac{2x^2 - 14x + 20}{7x - 2x^2 - 6}$

19. $\dfrac{2(x^2 - y^2)xy + x^4 - y^4}{x^4 - y^4}$

20. $\dfrac{y^6 + 64}{y^4 - 4y^2 + 16}$

21. $\dfrac{4a^2 - 1}{12a^2 + a - 4a^3 - 3}$

22. $\dfrac{a^2 - 2ab + 3b^2}{a^4 + 2a^2b^2 + 9b^4}$

23. $\dfrac{(x^2 - 16)(x^2 - 4x + 16)}{x^3 + 64}$

24. $\dfrac{15ab - 20a - 21b + 28}{21 - a - 10a^2}$

3-7 ADDITION OF FRACTIONS

The algebraic sum of two or more fractions having the same denominator is a fraction with the common denominator and a numerator which is the algebraic sum of the numerators of the fractions considered. This was proved in Problem 13, Article 2-4.

Illustration. $\dfrac{2x^2}{x - 4} - \dfrac{3x}{x - 4} + \dfrac{5}{x - 4} \equiv \dfrac{2x^2 - 3x + 5}{x - 4}.$

To find the algebraic sum of two or more fractions with different denominators, we must replace the fractions with equivalent fractions having the same denominators. It is preferable to use the *least common denominator* (LCD). The LCD of two or more fractions consists of the product of all the unique prime factors in the denominators, each with an exponent equal to the largest exponent with which the factor appears, and is really a result of the following important theorem.

Theorem 3-5. $\dfrac{a}{b} + \dfrac{c}{d} \equiv \dfrac{ad + bc}{bd} \qquad (b, d \neq 0).$

Proof. We have

$$\frac{a}{b} + \frac{c}{d} \equiv \frac{ad}{bd} + \frac{bc}{bd},$$

by Theorem 2-8. If we now use Problem 13, Article 2-4, we have

$$\frac{ad}{bd} + \frac{bc}{bd} \equiv \frac{ad + bc}{bd},$$

which is our required result.

Example 1. Find the LCD of the fractions

$$\frac{3x}{x^2 - 4x + 4}, \quad \frac{5x^2}{3(x^2 - 4)}, \quad \frac{2}{2x^2 - x - 6}.$$

Solution. Factoring each denominator, we have

$$x^2 - 4x + 4 \equiv (x - 2)^2,$$
$$3(x^2 - 4) \equiv 3(x + 2)(x - 2),$$
$$2x^2 - x - 6 \equiv (2x + 3)(x - 2).$$

The LCD is $3(x + 2)(x - 2)^2(2x + 3)$.

After the LCD has been determined, equivalent fractions may be formed. Divide the LCD of a given fraction by the denominator of that fraction, and then multiply both numerator and denominator of the given fraction by the result. The equivalent fractions may now be added, as in the illustration above.

Example 2. Change the following fractions to equivalent ones, with their LCD as denominator, and find their sum.

$$\frac{4}{x+2}, \quad \frac{x+3}{x^2-4}, \quad \frac{2x+1}{x-2}.$$

Solution. The LCD is $(x + 2)(x - 2)$. Therefore,

$$\frac{4}{x+2} \equiv \frac{4(x-2)}{(x+2)(x-2)}, \quad \frac{x+3}{x^2-4} \equiv \frac{x+3}{(x+2)(x-2)},$$
$$\frac{2x+1}{x-2} \equiv \frac{(2x+1)(x+2)}{(x+2)(x-2)},$$

and

$$\frac{4}{x+2} + \frac{x+3}{x^2-4} + \frac{2x+1}{x-2}$$
$$\equiv \frac{4(x-2)}{(x+2)(x-2)} + \frac{x+3}{(x+2)(x-2)} + \frac{(2x+1)(x+2)}{(x+2)(x-2)}$$
$$\equiv \frac{(4x-8) + (x+3) + (2x^2+5x+2)}{x^2-4}$$
$$\equiv \frac{2x^2+10x-3}{x^2-4}.$$

PROBLEMS

Reduce the following to single fractions and simplify.

1. $\frac{2}{3} + \frac{5}{6} - \frac{3}{10}$
2. $5 - \frac{4}{9} - \frac{7}{15}$
3. $\frac{3x}{4y} - \frac{4y}{3x}$
4. $\frac{a^2}{b} - \frac{b^2}{a}$
5. $\frac{2x+3}{6} - \frac{4x-7}{9}$
6. $\frac{3x-1}{5} + \frac{4-5x}{6}$

7. $x + y + \dfrac{x^2}{x - y}$

8. $\dfrac{x + 1}{x + 2} - \dfrac{x + 3}{x}$

9. $\dfrac{3x - 2y}{5x - 3} + \dfrac{2x - y}{3 - 5x}$

10. $\dfrac{2}{12x^2 - 3} + \dfrac{3}{2x - 4x^2}$

11. $\dfrac{5}{x} - \dfrac{4}{y} + \dfrac{3}{z}$

12. $\dfrac{4}{x^2 - 4x - 5} + \dfrac{2}{x^2 - 1}$

13. $\dfrac{2x - 1}{4 - x} + \dfrac{x + 2}{3x - 12}$

14. $\dfrac{x + 5}{x^2 + 7x + 10} - \dfrac{x - 1}{x^2 + 5x + 6}$

15. $\dfrac{x - 1}{2x^2 - 13x + 15} + \dfrac{x + 3}{2x^2 - 15x + 18}$

16. $\dfrac{2x + 3}{3x^2 + x - 2} - \dfrac{3x - 4}{2x^2 - 3x - 5}$

17. $\dfrac{3}{a - 3} + \dfrac{a^2 + 2}{a^3 - 27}$

18. $\dfrac{2xy}{x^3 + y^3} - \dfrac{x}{x^2 - xy + y^2}$

19. $\dfrac{2}{x^2 + 3x + 2} - \dfrac{3}{x^2 + 5x + 6} - \dfrac{4}{x^2 + 4x + 3}$

20. $x + 6 + \dfrac{5x + 1}{12x^2 + 5x - 2} - \dfrac{x}{3x + 2}$

21. $2y - 3 + \dfrac{y - 2}{4y^2 - 12y + 9} + \dfrac{y + 2}{2y^2 - y - 3}$

22. $\dfrac{1}{(x - y)(y - z)} + \dfrac{1}{(y - z)(z - x)} + \dfrac{1}{(z - x)(x - y)}$

23. $\dfrac{x}{(x - y)(y - z)} + \dfrac{y}{(y - z)(z - x)} + \dfrac{z}{(z - x)(x - y)}$

24. $\dfrac{2x - 1}{2x^2 - x - 6} + \dfrac{x + 3}{6x^2 + x - 12} - \dfrac{2x - 3}{3x^2 - 10x + 8}$

3-8 MULTIPLICATION AND DIVISION OF FRACTIONS

In algebra, as in arithmetic, the product of two or more fractions is a fraction whose numerator is the product of all numerators and whose denominator is the product of all denominators. In obtaining these products, the process of dividing out factors common to the numerator and denominator may be used. Results should be reduced to their simplest form. Note in Illustration 1 that this method is a direct application of Theorem 2-7 followed by an application of Theorem 2-8.

Illustration 1. $\dfrac{x - 4}{2x + 8} \cdot \dfrac{4x + 8}{x^2 - 16} \equiv \dfrac{(x - 4) \cdot 2^2 \cdot (x + 2)}{2(x + 4)(x + 4)(x - 4)}$

$$\equiv \frac{2(x + 2)}{(x + 4)^2}.$$

Algebra Sample in Nemeth Code

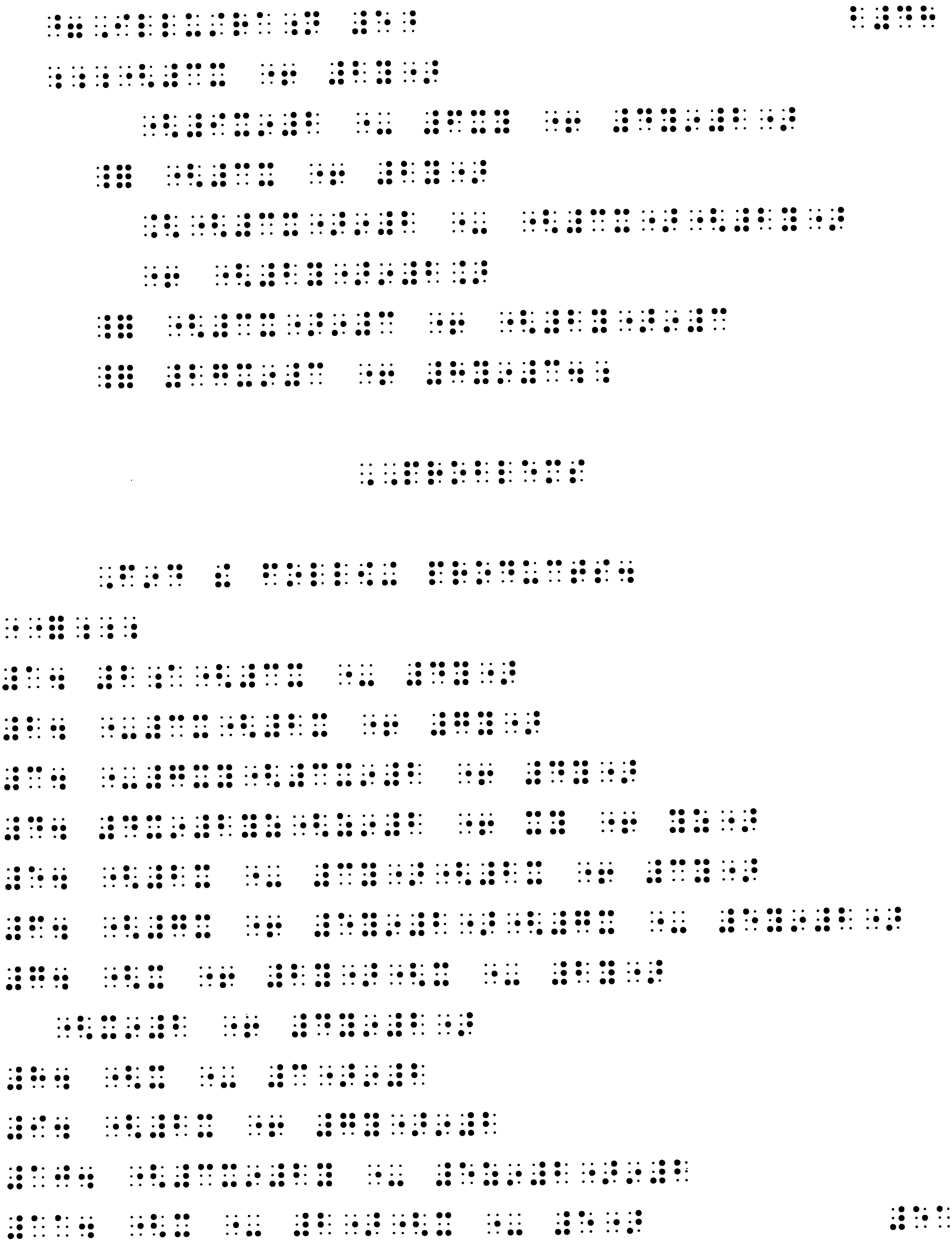
Nemeth Symbols

⠸	punctuation indicator	⠤	minus sign
⠘	begin superscript	⠬⠤	plus or minus sign
⠐	return to baseline after superscript	⠤⠬	minus or plus sign
⠹	begin fraction	⠡	times sign (dot)
⠌	horizontal fraction line	⠸⠌	slash
⠼	end fraction	⠸⠇	equivalent sign (3 horizontal bars)
⠷	opening parenthesis	⠨⠅	equal sign
⠾	closing parenthesis	⠌⠨⠅	not equal sign
⠈⠷	opening bracket	⠈⠼	asterisk
⠈⠾	closing bracket	⠠	mathematical comma
⠬	plus sign		

Algebra Sample in UEBC

UEBC Symbols

⠰	grade one symbol
⠰⠰⠰	begin grade one passage
⠰	(before a space) end grade one passage
⠐⠐⠿⠰⠰⠰	(on a line by itself) begin grade one passage
⠐⠐⠿⠰	(on a line by itself) end grade one passage
⠠⠠⠠	begin capitalized passage
⠠	(before a space) end capitalized passage
⠨⠂	italic word
⠨⠶	begin italic passage
⠨	(before a space) end italic passage
⠘⠂	bold word
⠘⠶	begin bold passage
⠘	(before a space) end bold passage
⠷	begin fraction
⠨⠌	fraction line
⠾	end fraction
⠔	superscript next item
⠣	begin compound item
⠜	end compound item
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠨⠣	opening bracket
⠨⠜	closing bracket
⠲	decimal point
⠐⠖	plus sign
⠐⠤	minus sign
⠸⠖	plus or minus sign
⠸⠤	minus or plus sign
⠐⠲	times sign (dot)
⠐⠶	equal sign
⠐⠶ ⠈⠱	not equal sign
⠸⠿	equivalent sign (3 horizontal bars)
⠸⠌	slash
⠐⠔	asterisk



Sample 3. Calculus

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).

5

Vector Integral Calculus

Part I. Two-Dimensional Theory

5.1 INTRODUCTION

The topic of this chapter is *line and surface integrals*. It will be seen that these can both be regarded as integrals of vectors and that the principal theorems can be most simply stated in terms of vectors; hence the title "vector integral calculus."

A familiar line integral is that of arc length: $\int_C ds$. The subscript C indicates that one is measuring the length of a curve C, as in Fig. 5.1. If C is given in parametric form $x = x(t)$, $y = y(t)$, the line integral reduces to the ordinary definite integral:

$$\int_C ds = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt.$$

If the curve C represents a wire whose density (mass per unit length) varies along C, then the wire has a total mass

$$M = \int_C f(x, y)\, ds,$$

where $f(x, y)$ is the density at the point (x, y) of the wire. The new integral can be expressed in terms of a parameter as previously or can be thought of simply as a limit of a sum

$$\int_C f(x, y)\, ds = \lim \sum_{i=1}^{n} f(x_i^*, y_i^*)\, \Delta_i s.$$

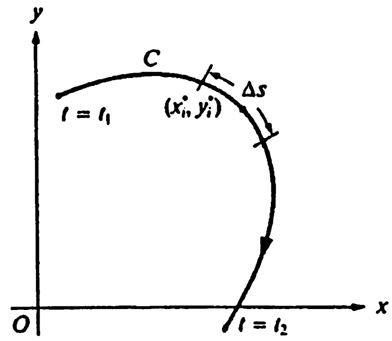


Figure 5.1 Line integral.

Here the curve has been subdivided into n pieces of lengths $\Delta_1 s, \Delta_2 s, \ldots, \Delta_n s$, and the point (x_i^*, y_i^*) lies on the ith piece. The limit is taken as n becomes infinite, while the maximum $\Delta_i s$ approaches 0.

A third example of a line integral is that of *work*. If a particle moves from one end of C to the other under the influence of a force $\mathbf{F}$, the work done by this force is defined as

$$\int_C F_T ds,$$

where F_T denotes the component of $\mathbf{F}$ on the tangent $\mathbf{T}$ in the direction of motion. This integral can be thought of as a limit of a sum as previously. However, another interpretation is possible. We first remark that the work done by a constant force $\mathbf{F}$ in moving a particle from A to B on the line segment AB is $\mathbf{F} \cdot \overrightarrow{AB}$; for this scalar product is equal to $|\mathbf{F}| \cdot \cos\alpha \cdot |\overrightarrow{AB}|$, α being the angle between $\mathbf{F}$ and $\overrightarrow{AB}$, and hence to the product of force component in direction of motion by the distance moved. Now the motion of the particle along C can be thought of as the sum of many small displacements along line segments, as suggested in Fig. 5.2. If these displacements are denoted by $\Delta_1\mathbf{r}, \Delta_2\mathbf{r}, \ldots, \Delta_n\mathbf{r}$, the work done would be approximated by a sum of form

$$\sum_{i=1}^{n} \mathbf{F}_i \cdot \Delta_i\mathbf{r},$$

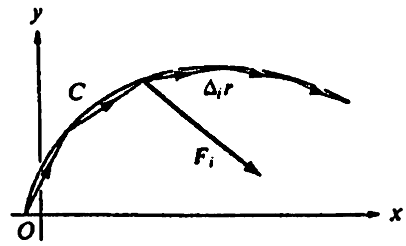


Figure 5.2 Work $= \int_C \mathbf{F} \cdot d\mathbf{r}$.

where $\mathbf{F}_i$ is the force acting for the ith displacement. The limiting form of this is again equal to the line integral $\int F_T ds$, but because of the way the limit is obtained, we can also write it as

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

One can thus write

$$\text{work} = \int_C F_T ds = \int_C \mathbf{F} \cdot d\mathbf{r}.$$

If the displacement vector $\Delta\mathbf{r}$ and force $\mathbf{F}$ are expressed in components,

$$\mathbf{F} = F_x\mathbf{i} + F_y\mathbf{j}, \qquad \Delta\mathbf{r} = \Delta x\mathbf{i} + \Delta y\mathbf{j},$$

the element of work $\mathbf{F} \cdot \Delta\mathbf{r}$ becomes

$$\mathbf{F} \cdot \Delta\mathbf{r} = F_x \Delta x + F_y \Delta y.$$

The total amount of work done is then approximated by a sum of form

$$\sum (F_x \Delta x + F_y \Delta y) = \sum F_x \Delta x + \sum F_y \Delta y.$$

The limiting form of this is a sum of two integrals:

$$\int_C F_x\, dx + \int_C F_y\, dy.$$

The first integral represents the work done by the x-component of the force; the second integral represents the work done by the y-component of the force.

It thus appears that one has three types of line integrals to consider, namely, the types

$$\int_C f(x, y)\, ds, \qquad \int_C P(x, y)\, dx, \qquad \int_C Q(x, y)\, dy,$$

which are limits of sums

$$\sum f(x, y)\, \Delta s, \qquad \sum P(x, y)\, \Delta x, \qquad \sum Q(x, y)\, \Delta y.$$

The foregoing gives the basis for the theory of line integrals in the plane. A very slight extension of these ideas leads to line integrals in space:

$$\int_C f(x, y, z)\, ds, \qquad \int_C f(x, y, z)\, dx, \qquad \ldots .$$

Surface integrals appear as a natural generalization, with the surface area element $d\sigma$ replacing the arc element ds:

$$\iint_S f(x, y, z)\, d\sigma = \lim \sum f(x, y, z)\, \Delta\sigma.$$

There are corresponding component integrals

$$\iint_S f(x, y, z)\, dx\, dy, \qquad \iint_S f(x, y, z)\, dy\, dz, \qquad \ldots$$

and a vector surface integral

$$\iint_S \mathbf{F} \cdot d\boldsymbol{\sigma} = \iint_S (\mathbf{F} \cdot \mathbf{n})\, d\sigma,$$

where $d\boldsymbol{\sigma} = \mathbf{n}\, d\sigma$ is the "area element vector," $\mathbf{n}$ being a unit normal vector to the surface.

It will be seen that the basic theorems—those of Green, Gauss, and Stokes—concern the relations between line, surface, and volume (triple) integrals. These correspond to fundamental physical relations between such quantities as flux, circulation, divergence, and curl. The applications will be considered at the end of the chapter.

5.2 LINE INTEGRALS IN THE PLANE

We now state in precise form the definitions outlined in the preceding section.

By a *smooth curve* C in the xy-plane will be meant a curve representable in the form:

$$x = \phi(t), \qquad y = \psi(t), \qquad h \leqq t \leqq k, \tag{5.1}$$

where x and y are continuous and have continuous derivatives for $h \leqq t \leqq k$. The curve C can be assigned a direction, which will usually be that of increasing t. If A denotes the point $[\phi(h), \psi(h)]$ and B denotes the point $[\phi(k), \psi(k)]$, then C can be thought of as the path of a point moving continuously from A to B. This path may cross itself, as for the curve C_1 of Fig. 5.3. If the initial point A and terminal point B coincide, C is termed a *closed* curve; if, in addition, (x, y) moves from A to $B = A$ without retracing any other point, C is called a *simple closed* curve (curve C_2 of Fig. 5.3).

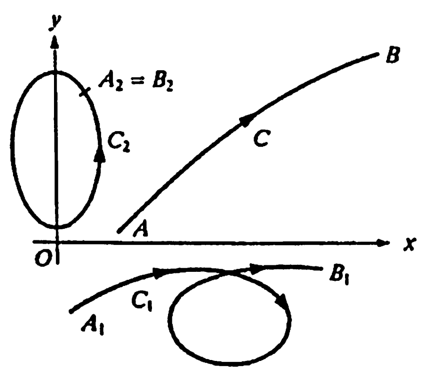


Figure 5.3 Paths of integration.

Let C be a smooth curve as previously, with positive direction that of increasing t. Let $f(x, y)$ be a function defined at least when (x, y) is on C. The line integral $\int_C f(x, y)\, dx$ is defined as a limit:

$$\int_C f(x, y)\, dx = \lim \sum_{i=1}^{n} f(x_i^*, y_i^*)\, \Delta_i x. \tag{5.2}$$

The limit refers to a subdivision of C as indicated in Fig. 5.4. The successive subdivision points are A: (x_0, y_0), $(x_1, y_1), \ldots, B$: (x_n, y_n). These correspond to parameter values: $h = t_0 < t_1 < \cdots < t_n = k$. The point (x_i^*, y_i^*) is some point of C between (x_{i-1}, y_{i-1}) and (x_i, y_i); that is, (x_i^*, y_i^*) corresponds to a parameter value t_i^*, where $t_{i-1} \leq t_i^* \leq t_i$. $\Delta_i x$ denotes the difference $x_i - x_{i-1}$. The limit is taken as n becomes infinite and the largest $\Delta_i t$ approaches 0, where $\Delta_i t = t_i - t_{i-1}$. Similarly,

$$\int_C f(x, y)\, dy = \lim \sum f(x_i^*, y_i^*)\, \Delta_i y, \tag{5.3}$$

where $\Delta_i y = y_i - y_{i-1}$.

The significance of these definitions is guaranteed by the following basic theorems:

I If $f(x, y)$ is continuous on C, then

$$\int_C f(x, y)\, dx \quad \text{and} \quad \int_C f(x, y)\, dy \quad \text{exist.}$$

II If $f(x, y)$ is continuous on C, then

$$\int_C f(x, y)\, dx = \int_h^k f[\phi(t), \psi(t)]\, \phi'(t)\, dt, \tag{5.4}$$

$$\int_C f(x, y)\, dy = \int_h^k f[\phi(t), \psi(t)]\, \psi'(t)\, dt. \tag{5.5}$$

Formulas (5.4) and (5.5) reduce the integrals to ordinary definite integrals and are thus essential for computation of particular integrals. Thus let C be the

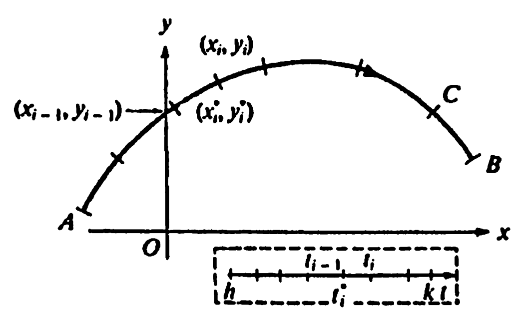


Figure 5.4 Definition of line integral.

path $x = 1 + t, y = t^2, 0 \leq t \leq 1$, directed with increasing t. Then

$$\int_C (x^2 - y^2)\,dx = \int_0^1 \left[(1+t)^2 - t^4\right] dt = \tfrac{32}{15},$$

$$\int_C (x^2 - y^2)\,dy = \int_0^1 \left[(1+t)^2 - t^4\right] 2t\,dt = 2\tfrac{1}{2}.$$

It is logically easier to prove II first, for I is an immediate consequence of II. To prove II, one notes that the sum $\Sigma f(x_i^*, y_i^*)\,\Delta_i x$ can be written as

$$\sum_{i=1}^{n} f[\phi(t_i^*), \psi(t_i^*)] \frac{\Delta_i x}{\Delta_i t} \Delta_i t.$$

Now $\Delta_i x = x_i - x_{i-1} = \phi'(t_i^{**})\,\Delta_i t$ by the Law of the Mean. Hence the sum can be written as

$$\sum_{i=1}^{n} F(t_i^*)\phi'(t_i^{**})\,\Delta_i t,$$

where $F(t) = f[\phi(t), \psi(t)]$ and t_i^* and t_i^{**} are both between t_{i-1} and t_i. It is easily shown [see CLA, Section 12-25] that this sum approaches as limit the integral

$$\int_h^k F(t)\phi'(t)\,dt = \int_h^k f[\phi(t), \psi(t)]\phi'(t)\,dt$$

as required. Formula (5.5) is proved in the same way.

We remark that the value of a line integral on C does not depend on the particular parametrization of C, but only on the order in which the points of C are traced. (See Problem 5.)

In many applications the path C is not itself smooth but is composed of a finite number of arcs, each of which is smooth. Thus C might be a broken line. In this case, C is termed *piecewise* smooth. The line integral along C is simply, by definition, the sum of the integrals along the pieces. One verifies at once that (5.2), (5.3), and the theorems I and II continue to hold. In (5.4) and (5.5) the functions $\phi'(t)$ and $\psi'(t)$ will have jump discontinuities, which will not interfere with the existence of the integral (cf. Section 4.1). *Throughout this book all paths of integration for line integrals will be piecewise smooth unless otherwise specified.*

If the curve C is represented in the form

$$y = g(x), \qquad a \leqq x \leqq b,$$

then one can regard x itself as parameter, replacing t; that is, C is given by the equations

$$x = x, \qquad y = g(x), \qquad a \leqq x \leqq b$$

in terms of the parameter x. If the direction of C is that of increasing x, (5.4) and

(5.5) become

$$\int_C f(x, y)\, dx = \int_a^b f[x, g(x)]\, dx, \tag{5.6}$$

$$\int_C f(x, y)\, dy = \int_a^b f[x, g(x)]\, g'(x)\, dx. \tag{5.7}$$

The ordinary definite integral $\int_a^b y\, dx$, where $y = g(x)$, is a special case of (5.6).

Similarly, if C is represented in the form

$$x = F(y), \qquad c \leqq y \leqq d,$$

and the direction of C is that of increasing y, then

$$\int_C f(x, y)\, dx = \int_c^d f[F(y), y]\, F'(y)\, dy, \tag{5.8}$$

$$\int_C f(x, y)\, dy = \int_c^d f[F(y), y]\, dy. \tag{5.9}$$

In most applications the line integrals appear as a combination,

$$\int_C P(x, y)\, dx + \int_C Q(x, y)\, dy,$$

which is abbreviated as follows:

$$\int_C [P(x, y)\, dx + Q(x, y)\, dy] \quad \text{or} \quad \int_C P(x, y)\, dx + Q(x, y)\, dy,$$

the brackets being used only when necessary.

In the formulas thus far the direction of C has been that of increasing parameter. If the opposite direction is chosen, upper and lower limits are reversed on all integrals. Thus (5.4) becomes

$$\int_C f(x, y)\, dx = \int_k^h f[\phi(t), \psi(t)]\, \phi'(t)\, dt. \tag{5.4'}$$

The line integral is therefore multiplied by -1. Often it is convenient to specify the path by its equations in some form and to indicate the direction by using the initial and terminal points as lower and upper limits:

$$\int_{C\ A}^{B} P\, dx + Q\, dy \quad \text{or} \quad \int_{C\ (x_1, y_1)}^{(x_2, y_2)} P\, dx + Q\, dy.$$

It will be seen later that under certain conditions, one needs only prescribe initial and terminal points:

$$\int_A^B P\, dx + Q\, dy.$$

EXAMPLE 1 To evaluate

$$\int_{C\ (1,0)}^{(-1,0)} (x^3 - y^3)\, dy,$$

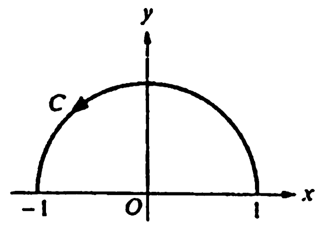


Figure 5.5 Example 1.

where C is the semicircle $y = \sqrt{1 - x^2}$ shown in Fig. 5.5, one can represent C parametrically:

$$x = \cos t, \qquad y = \sin t, \qquad 0 \leqq t \leqq \pi,$$

and the integral becomes

$$\int_0^{\pi} (\cos^3 t - \sin^3 t) \cos t \, dt = \frac{3\pi}{8}.$$

One can use x as parameter, and the integral becomes

$$\int_1^{-1} \left[x^3(1 - x^2)^{\frac{3}{2}} \right] \frac{-x}{\sqrt{1 - x^2}} dx;$$

this is clearly in a more awkward form for integration. The substitution $x = \cos t$ brings one back to the parametric form. One can use y as parameter but has then to split the integral into two parts, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(-1, 0)$;

$$\int_0^1 \left[(1 - y^2)^{\frac{3}{2}} - y^3 \right] dy + \int_1^0 \left[-(1 - y^2)^{\frac{3}{2}} - y^3 \right] dy = 2\int_0^1 (1 - y^2)^{\frac{3}{2}} dy.$$

Note that $x = \sqrt{1 - y^2}$ on the first part of the path and $x = -\sqrt{1 - y^2}$ on the second part. ■

EXAMPLE 2 Let C be the parabolic arc $y = x^2$ from $(0, 0)$ to $(-1, 1)$. Then

$$\int_C xy^2 \, dx + x^2 y \, dy = \int_0^{-1} \left(xy^2 + x^2 y \frac{dy}{dx} \right) dx = \int_0^{-1} (x^5 + 2x^5) \, dx = \tfrac{1}{2}. \quad ■$$

If C is a *closed* curve, then there is no need to specify initial and terminal point, though the direction must be indicated. If C is a simple closed curve (traced just once), then one need only specify which of the two possible directions is chosen. The notations

$$\text{(a)} \ \oint\limits^{\circlearrowleft} P \, dx + Q \, dy, \qquad \text{(b)} \ \oint\limits^{\circlearrowright} P \, dx + Q \, dy$$

refer to the two cases of Figs. 5.6(a) and 5.6(b). The counterclockwise arrow refers to what is roughly a counterclockwise direction on C; this will be termed the *positive* direction (as for angular measure); the clockwise direction will be called

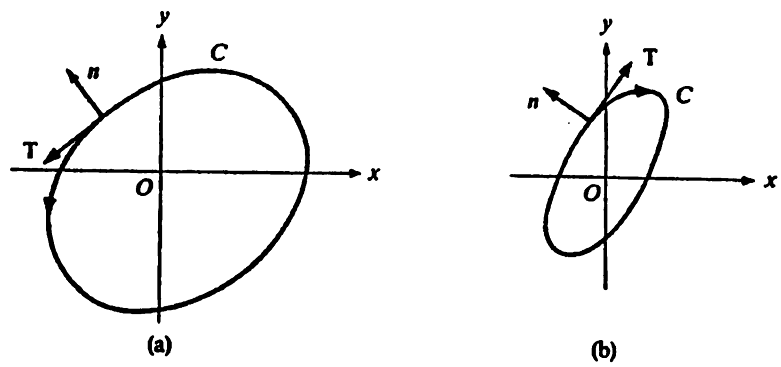


Figure 5.6 (a) Positive direction and (b) negative direction on a simple closed path.

the *negative* direction. It should be noted that the direction can be specified by reference to the unit tangent vector **T** in the direction of integration and the unit normal vector **n** that points to the outside of the region bounded by C; for the positive direction, **n** is 90° behind **T**, as in Fig. 5.6(a); for the negative direction, **n** is 90° ahead of **T** as in Fig. 5.6(b).

EXAMPLE 3 To evaluate

$$\oint_C y^2\,dx + x^2\,dy,$$

where C is the triangle with vertices $(1,0)$, $(1,1)$, $(0,0)$, shown in Fig. 5.7, one has to compute three integrals. The first is the integral from $(0,0)$ to $(1,0)$; along this path, $y = 0$ and, if x is the parameter, $dy = 0$. Hence the first integral is 0. The second integral is that from $(1,0)$ to $(1,1)$; if y is used as parameter, this reduces to

$$\int_0^1 dy = 1,$$

since $dx = 0$. For the third integral, from $(1,1)$ to $(0,0)$, x can be used as parameter, so that the integral is

$$\int_1^0 2x^2\,dx = -\tfrac{2}{3},$$

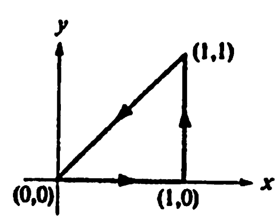


Figure 5.7 Example 3.

since $dy = dx$. Thus finally

$$\oint_C y^2\,dx + x^2\,dy = 0 + 1 - \tfrac{2}{3} = \tfrac{1}{3}. \quad \blacksquare$$

5.3 INTEGRALS WITH RESPECT TO ARC LENGTH • BASIC PROPERTIES OF LINE INTEGRALS

For a smooth or piecewise smooth path C, as in the preceding section, arc length s is well defined. Thus s can be defined as the distance traversed from the initial point ($t = h$) up to a general t:

$$s = \int_h^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\,dt. \tag{5.10}$$

If the curve C is directed with increasing t, then s also increases in the direction of motion, going from 0 up to the length L of C. Let C be subdivided as in Fig. 5.4 and let $\Delta_i s$ denote the increment in s from t_{i-1} to t_i, that is, the distance moved in this interval. One then makes the definition

$$\int_C f(x, y)\,ds = \lim_{\substack{n\to\infty \\ \max \Delta_i s \to 0}} \sum_{i=1}^{n} f(x_i^*, y_i^*)\,\Delta_i s. \tag{5.11}$$

If f is continuous on C, this integral will exist and can be evaluated as follows:

$$\int_C f(x, y)\,ds = \int_h^k f[\phi(t), \psi(t)]\sqrt{\phi'(t)^2 + \psi'(t)^2}\,dt. \tag{5.12}$$

This is proved in the same way as (5.4) and (5.5), with the aid of the formula

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\phi'(t)^2 + \psi'(t)^2}.$$

One can in principle use s itself as the parameter on the curve C; if this is done, x and y become functions of s: $x = x(s)$, $y = y(s)$. The point $[x(s), y(s)]$ is then the position of the moving point after a distance s has been traversed. In this case, (5.11) reduces to a definite integral with respect to s:

$$\int_C f(x, y)\,ds = \int_0^L f[x(s), y(s)]\,ds. \tag{5.13}$$

If x is used as parameter, one has

$$\int_C f(x, y)\,ds = \int_a^b f[x, y(x)]\sqrt{1 + \left(\frac{dy}{dx}\right)^2}\,dx; \tag{5.14}$$

there is an analogous formula for y.

The basic combination

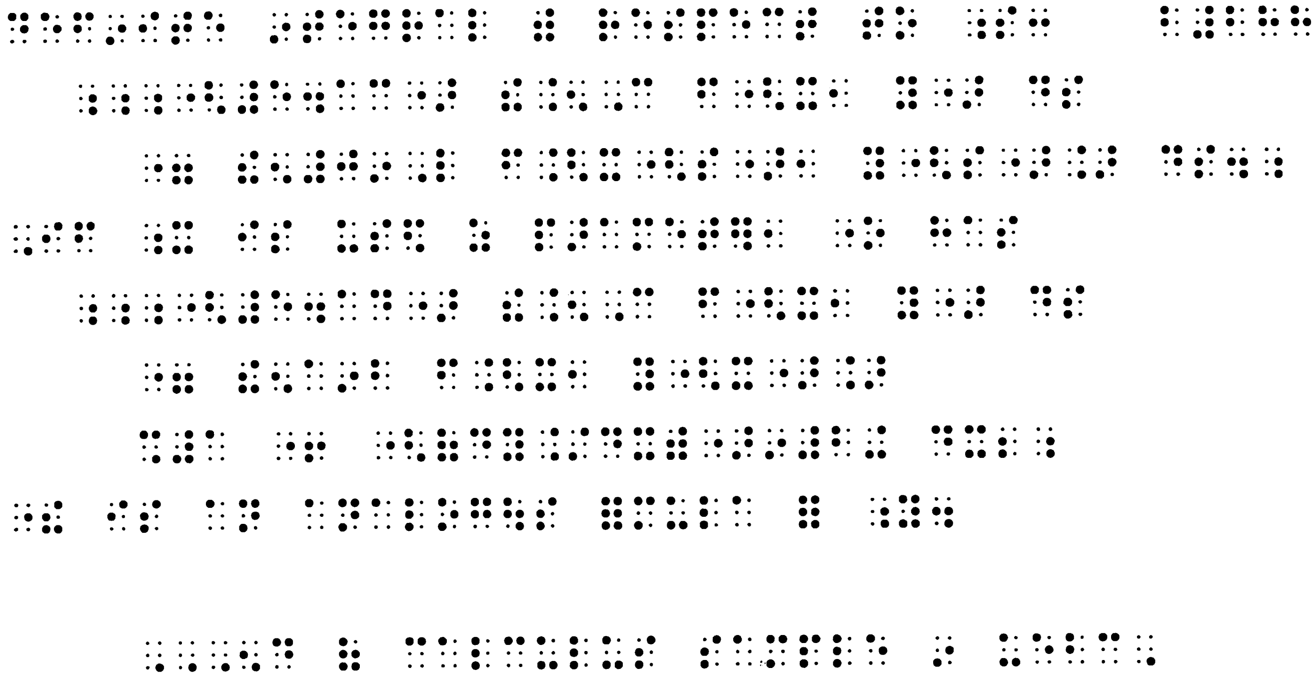
$$\int_C P\,dx + Q\,dy$$

Nemeth Symbols

⠸	punctuation indicator
⠸	boldface letter indicator
⠐	begin modified expression
⠩	begin modifier below
⠣	begin modifier above
⠻	end modified expression
⠰	begin subscript
⠰⠰	begin subscript within subscript
⠘	begin superscript
⠘⠰	begin subscript within superscript
⠪	comma within subscript or superscript
⠐	return to baseline after subscript or superscript
⠜	begin square root
⠻	end square root
⠹	begin fraction
⠸⠹	begin fraction portion of mixed number
⠌	horizontal fraction line
⠼	end fraction
⠸⠼	end fraction portion of mixed number
⠠	mathematical comma
⠨	decimal point
⠬	plus sign
⠤	minus sign
⠡	times sign (dot)
⠨⠅	equal sign
⠐⠅	less than sign
⠐⠅ ⠨⠅	less than or equal sign
⠷	opening parenthesis
⠾	closing parenthesis
⠈⠷	opening bracket
⠈⠾	closing bracket
⠈⠼	asterisk
⠮	integral sign
⠮⠈⠫⠉⠫⠢⠔⠕⠻	integral sign with superimposed circle bearing counterclockwise arrow
⠮⠈⠫⠉⠫⠪⠢⠔⠻	integral sign with superimposed circle bearing clockwise arrow
⠠⠿	infinity sign
⠄	prime sign
⠘⠨⠡	degree sign
⠫⠕	right arrow
⠳	vertical bar
⠫⠸⠲	solid square
⠨⠠⠙	Greek capital delta
⠨⠠⠎	Greek capital sigma
⠨⠁	Greek alpha
⠨⠋	Greek phi
⠨⠏	Greek pi
⠨⠽	Greek psi
⠨⠎	Greek sigma

UEBC Symbols

Symbol	Meaning
⠰	grade one symbol
⠰⠰	grade one word
⠰⠰⠰	begin grade one passage
⠰	(before a space) end grade one passage
⠐⠐⠿ ⠰⠰⠰	(on a line by itself) begin grade one passage
⠐⠐⠿ ⠰	(on a line by itself) end grade one passage
⠠⠠⠠	begin capitalized passage
⠠	(before a space) end capitalized passage
⠨⠂	italic word
⠨⠶	begin italic passage
⠨	(before a space) end italic passage
⠘⠆	bold letter
⠘⠂	bold word
⠷	begin fraction
⠨⠌	fraction line
⠾	end fraction
⠣	begin compound item
⠜	end compound item
⠩	begin square root
⠬	end square root
⠔	superscript next item
⠢	subscript next item
⠨⠢	next item directly below previous item
⠨⠔	next item directly above previous item
⠯	superimpose next symbol on previous symbol
⠨⠱	arrow over previous item
⠲	decimal point
⠲⠲⠲	ellipsis
⠐⠖	plus sign
⠐⠤	minus sign
⠐⠲	times sign (dot)
⠐⠶	equal sign
⠈⠣	less than sign
⠸⠈⠣	less than or equal sign
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠨⠣	opening bracket
⠨⠜	closing bracket
⠠⠤	dash
⠐⠔	asterisk
⠮	integral sign
⠼⠿	infinity sign
⠶	prime sign
⠘⠚	degree sign
⠳⠕	right arrow
⠸⠳	vertical line
⠸⠫⠼⠙	filled square
⠈⠫⠉⠉	transcriber-defined shape: circle with counterclockwise arrow
⠈⠫⠉⠇	transcriber-defined shape: circle with clockwise arrow
⠱	end shape
⠠⠨⠙	capital Greek delta
⠠⠨⠎	capital Greek sigma
⠨⠁	Greek alpha
⠨⠋	Greek phi
⠨⠏	Greek pi
⠨⠽	Greek psi
⠨⠎	Greek sigma



Sample 4. Teaching Material

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

Teaching Material in Nemeth Code

Nemeth Symbols

⠸	punctuation indicator
⠿	general omission symbol
⠐	begin modified expression
⠣	begin modifier above
⠻	end modified expression
⠸	bold next letter
⠬	plus sign
⠤	minus sign
⠈⠡	times sign (cross)
⠡	times sign (dot)
⠨⠌	divided by sign
⠕	divided by sign (spatial)
⠬⠤	plus or minus sign
⠨⠅	equal sign
⠌ ⠨⠅	not equal sign
⠈⠱ ⠨⠅	equal sign under tilde
⠐⠅	less than sign
⠨⠂	greater than sign
⠐⠅⠱	bar under less than
⠨⠂⠱	bar under greater than
⠹	begin fraction
⠼	end fraction
⠌	horizontal fraction line
⠸⠌	diagonal fraction line
⠸⠹	begin fraction portion of mixed number
⠸⠼	end fraction portion of mixed number
⠠⠹	begin complex fraction
⠠⠼	end complex fraction
⠠⠌	horizontal complex fraction line
⠷	opening parenthesis
⠾	closing parenthesis
⠈⠷	opening bracket
⠈⠾	closing bracket
⠨⠷	opening brace
⠨⠾	closing brace
⠳	vertical bar
⠈ ⠠⠷	enlarged opening bracket
⠈ ⠠⠾	enlarged closing bracket
⠨	decimal point
⠈⠴	percent sign
⠠	mathematical comma
⠈⠎	dollar sign
⠯	factorial sign
⠜	begin radical
⠻	end radical
⠣	index of radical indicator

Continued on next left-hand page.

Teaching Material in UEBC

UEBC Symbols

⠰	grade one symbol
⠰⠰	grade one word
⠰⠰⠰	begin grade one passage
⠰	(before a space) end grade one passage
⠐⠖	plus sign
⠐⠤	minus sign
⠐⠦	times sign (cross)
⠐⠲	times sign (dot)
⠐⠌	divided by sign
⠐⠜	divided by sign (spatial)
⠸⠖	plus or minus sign
⠐⠶	equal sign
⠈⠣	less than sign
⠈⠜	greater than sign
⠸⠈⠣	less than or equal sign
⠸⠈⠜	greater than or equal sign
⠐⠸⠔	tilde over equal sign
⠈⠔	tilde
⠨⠈⠣	much less than sign
⠌	numeric fraction line
⠷	begin fraction
⠾	end fraction
⠨⠌	fraction line
⠸⠌	slash
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠸⠳	vertical line
⠸⠣	opening brace
⠸⠜	closing brace
⠨⠣	opening bracket
⠨⠜	closing bracket
⠠⠨⠣	multi-line opening bracket
⠠⠨⠜	multi-line closing bracket
⠣	begin compound item
⠜	end compound item
⠲	decimal point
⠨⠴	percent sign
⠨⠎	dollar sign
⠖	factorial sign
⠈⠱	line through previous item
⠩	begin radical
⠩⠔	begin radical with next item as index of radical
⠬	end radical
⠔	superscript next item
⠢	subscript next item
⠨⠔	next item directly above previous item
⠐⠱	caret over previous item
⠱	bar over previous item
⠘⠱	right arrow over previous item

Continued on next right-hand page.

Nemeth Symbols (cont.)

⠘	begin superscript	⠱	horizontal bar
⠰	begin subscript	⠫⠇	parallel sign
⠪	comma within subscript	⠫⠏	perpendicular sign
⠐	return to baseline after superscript or subscript	⠘⠨⠡	degree sign
⠨⠼	crosshatch	⠐⠂	ratio sign
⠫⠪	angle sign	⠤⠤⠤⠤	double dash
⠸⠣	caret	⠈⠱	tilde
⠫⠁	arc	⠠⠿	infinity sign
⠫⠞	triangle	⠐⠅⠈⠐⠅⠻	two nested less than signs
⠫⠲	square	⠨⠠⠙	Greek capital delta
⠫⠉	circle	⠨⠛	Greek gamma
⠫⠪⠒⠒⠕	right and left barbed arrow	⠨⠠⠎	Greek capital sigma
⠫⠕	right arrow		

UEBC Symbols (cont.)

⠸⠹	crosshatch, or "number"
⠈⠫⠁	transcriber-defined shape: angle
⠈⠫⠗	transcriber-defined shape: arc
⠫⠼⠉	triangle
⠫⠼⠙	square
⠫⠿	circle
⠈⠫⠇	transcriber-defined shape: parallel sign
⠈⠫⠏	transcriber-defined shape: perpendicular sign
⠱	end shape
⠳⠕	right arrow
⠳⠺⠗⠕	left- and right-pointing arrow
⠘⠚	degree sign
⠒	(spaced) ratio sign
⠤⠤⠤⠤	blank (A UEBC symbol has not yet been assigned.)
⠘⠂	bold word
⠘⠆	bold symbol
⠠⠠⠠	begin capitalized passage
⠠⠄	end capitals within a sequence
⠠	(before a space) end capitalized passage
⠨⠶	begin italic passage
⠨	(before a space) end italic passage
⠼⠿	infinity sign
⠲⠲⠲	ellipsis
⠠⠨⠙	capital Greek delta
⠨⠛	Greek gamma
⠠⠨⠎	capital Greek sigma

Sample 4a. Math Workshop

A. Arithmetic

1. addition and subtraction

   $4 + 5 =$
   $15 - 8 =$

2. multiplication

   $29 \times 6 =$
   $453 \cdot 80 =$

3. division

   $55 \div 5 = 11$

   $24\overline{)9785}$

4. fractions

   $\frac{1}{2} \cdot \frac{1}{4} =$

   $\frac{5}{45} + \frac{6}{9} + \frac{7}{18} =$

5. mixed numbers

   $23\frac{1}{4} \times 2 =$

   $2\frac{1}{2} + 1\frac{3}{4} =$

6. decimals

   $45.6 + 2.4 =$
   $978.2 \div 12 =$

7. word problems (%, $)

   A suit that sold for $120 is on sale for $75. The percent of decrease is 37.5%.

8. comparison symbols <, >, =

   $5 = 8$ is false

B. Algebra

1. factorial and absolute value

   $5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$

   $5! = 120$

   $|-34.7|$

2. negative numbers

   $-4.5 - 16.3 =$
   $-12.2 + 3.7 =$

3. variables, linear equations, grouping symbols

   $y = 3x + 5$

   Simplify:
   $\{2x - 4[3x - 5(2x + 3) - 4x] + 6\}$

4. comparison symbols ≤, ≥, ≠, and the plus or minus sign ±

   $y \geq 2x - 3$

5. square roots and radicals with higher indices

   $\sqrt{237}$

   $\sqrt[3]{99}$

6. monomials, polynomials and quadratic equations

   $-4z^2$

   $(3x^2 + 2x - 1)(x^2 - 3)$

   $(x + 2)^2 =$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $3x^2 + 5x - 2 = 0$, then

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$$

7. rational expressions and complex fractions

$$\frac{4x - x^2}{2x + 3} \div \frac{16x^2 - 4x^3}{8x + 12}$$

$$\frac{\frac{1}{4} + \frac{2}{3}}{\frac{1}{2}} =$$

8. value table and matrix

2	4	6	8	10	12	x
8	16	24	32	40	?	?

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 19 \\ 1 & 26 \\ 0 & 33 \end{bmatrix}$$

9. table

Grading Period	# of A's
1	5
2	10
3	11

C. Geometry

1. angle and arc

$\angle ABC$

$\widehat{EF}$ $\widehat{EF}$

2. shapes

$\triangle ABC$

$\square ABCD$

$\bigcirc O$

3. line, line segment and ray

$\overleftrightarrow{XY}$ $\overline{AF}$ $\overrightarrow{AB}$

4. comparison symbols: $\parallel$, $\perp$, $\cong$

$\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

$\overline{AB} \perp \overline{CD}$

$\angle ABC \cong \angle CDE$

5. degrees

$90° + 90°$

⠎⠁⠇⠑ ⠿ ⠈⠎⠼⠛⠑⠲ ⠠⠮ ⠏⠻⠉⠢⠞ ⠷ ⠙⠑⠉⠗⠂⠎⠑
⠊⠎ ⠼⠉⠛⠲⠑⠨⠴⠲

⠼⠓⠲ ⠉⠕⠍⠏⠜⠊⠎⠕⠝ ⠎⠽⠍⠃⠕⠇⠎ ⠈⠣⠂ ⠈⠜⠂ ⠐⠶
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⠼⠉⠲ ⠧⠜⠊⠁⠃⠇⠑⠎⠂ ⠇⠔⠑⠜ ⠑⠟⠥⠁⠰⠝⠎⠂ ⠛⠗⠳⠏⠬
⠎⠽⠍⠃⠕⠇⠎
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⠠⠎⠊⠍⠏⠇⠊⠋⠽⠒
⠸⠣⠼⠃⠭ ⠐⠤ ⠼⠙⠨⠣⠼⠉⠭ ⠐⠤ ⠼⠑⠐⠣⠼⠃⠭ ⠐⠖ ⠼⠉⠐⠜
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⠮ ⠏⠇⠥⠎ ⠕⠗ ⠍⠔⠥⠎ ⠎⠊⠛⠝ ⠸⠖
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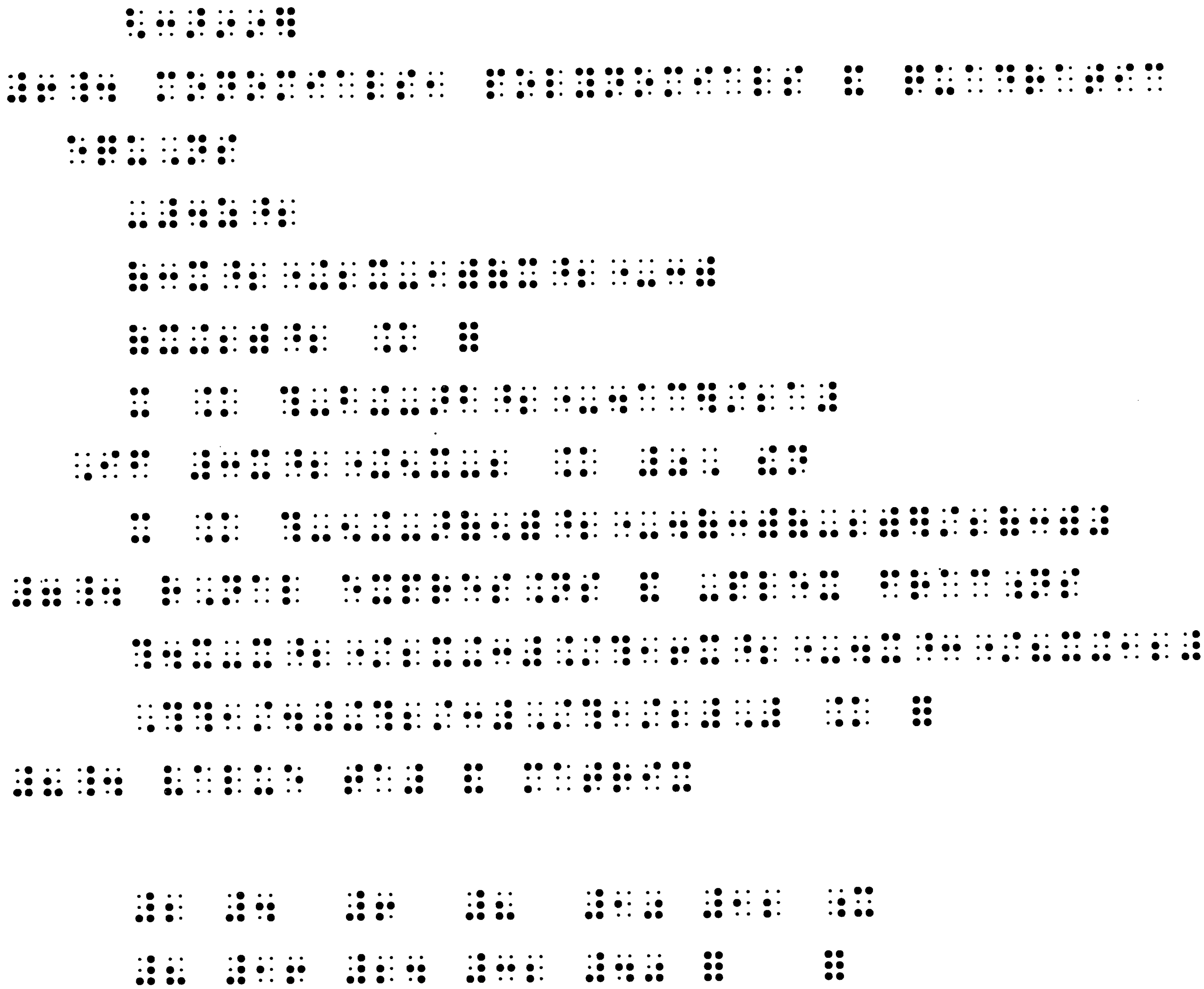
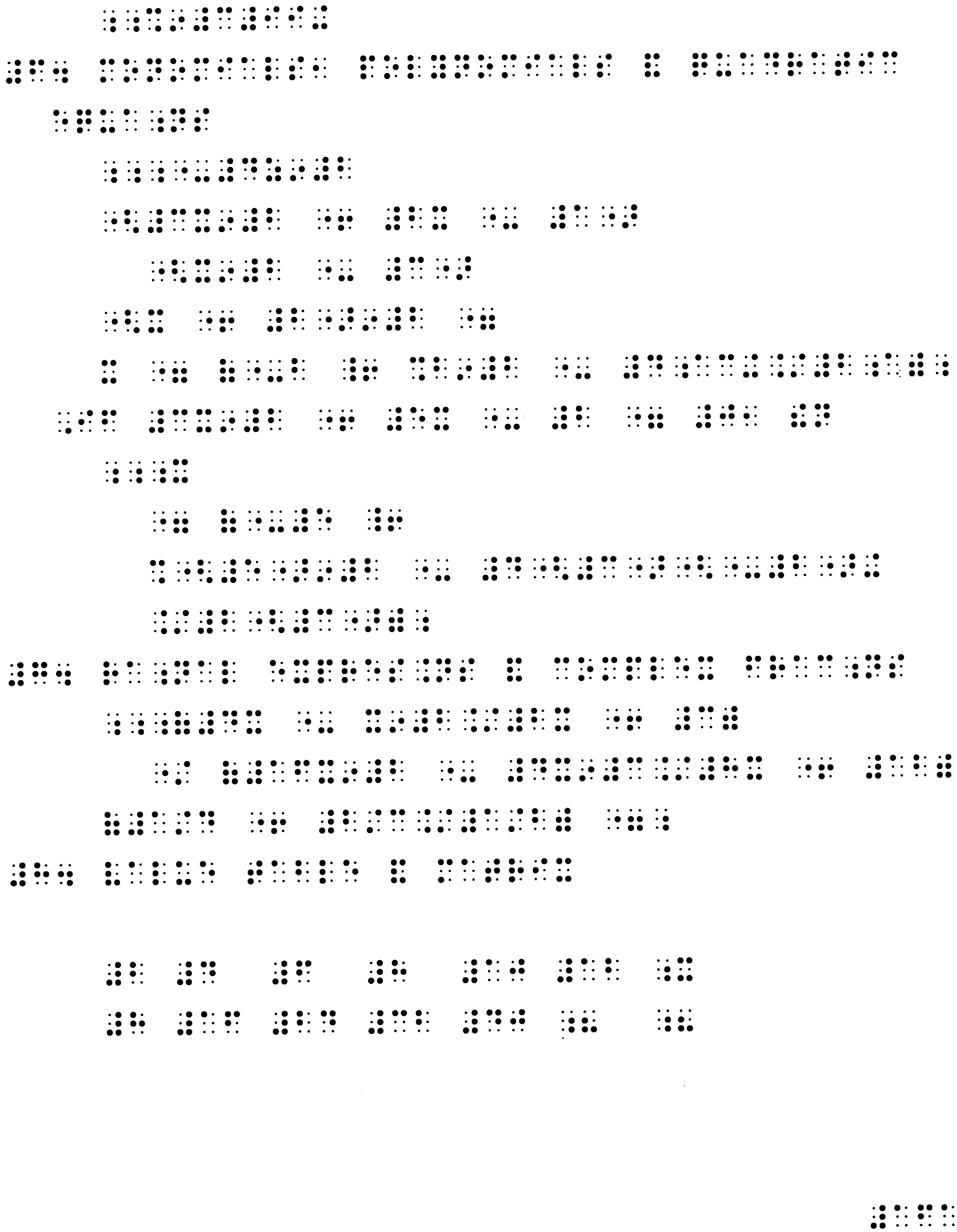




Sample 4b. Chapter 6 Review

Write each ratio in simplest form.

1. $\frac{AB}{CD}$
2. $\frac{m\angle BAC}{m\angle DAC}$
3. $\frac{BC}{AC}$
4. Determine whether the ratios $\frac{3}{5}$ and $\frac{15}{25}$ form a proportion.

Find the value of x in each proportion.

5. $14 : x = 21 : 27$
6. $\frac{5}{2} = \frac{15}{x}$
7. Complete the statement in five different ways.

   If $\frac{2}{3} = \frac{8}{12}$, then ________

8. Use the proportion $\frac{7}{8} = \frac{x}{y}$ to complete the statement.

   $\frac{x}{7} =$ ______

9. Write a two-column proof.

   Given: $\frac{JN}{NM} = \frac{JK}{KL}$

   Prove: $\frac{JM}{NM} = \frac{JL}{KL}$

10. $\overline{RS} \parallel \overline{UV}$ Find x and y.
11. Prove $\triangle EFG \sim \triangle EGH$.
12. Find the value of x for which the triangles are similar.
13. Given $\triangle TRS$ and $\triangle VWX$ with $\angle R \cong \angle W$, state the proportion that must be true if

    $\triangle TRS \sim \triangle VWX$ by SAS Similarity.

    Given $\triangle CAB$ and $a \parallel b \parallel c$, complete each statement and state the theorem applied.

14. If $CE = 3$, $CA = 9$, and $CB = 12$, then $CG =$ ______

Sample 4c. Algebra I Chapter 7 Test

1. Match the equation or inequality with the graph it describes.

   a. $|x| = 1$
   b. $|x| \le 1$
   c. $|x| \ge 1$

2. Which equation matches the graph?

   (*a*) $y = \left|\frac{1}{2}x\right| - 3$

   (*b*) $y = \left|\frac{1}{2}x + 1\right|$

   (*c*) $y = \left|\frac{1}{2}x\right| + 1$

   (*d*) $y = |x| + 1$

3. Solve the equation.

   $|x - 7| = 3$

4. Evaluate.

   $\sqrt{36}$

5. Complete the table. Then graph the equation $y = \sqrt{x} - 4$.

x	$\sqrt{x}$	$\sqrt{x} - 4$	(x, y)
0			
1			
4			
9			
16			

6. Find the taxi distance between (-3, 1) and (2, -4).

7. Use the Distance Formula to find the distance between (3, 5) and (-2, -3).

8. Use the proportion to find the value of *x*.

   $$\frac{x}{63} = \frac{5}{28}$$

9. The two triangles are similar. Find *x*, *y*, and *z*. (Hint: Begin by using the Pythagorean Theorem.)

10. For the triangle shown, find sin *K*, cos *K*, and tan *K*. Give exact answers.

11. A 17-ft ladder makes an angle of 65° with the ground. Give *AC*, the distance of the bottom of the ladder from the wall. Round your answer to the nearest tenth of a foot.

Sample 4d. Algebra II Chapter 3 Test

Solve each system of linear equations if possible. State the number of solutions, and tell whether the system is consistent or inconsistent.

1. $5x + 2y = -4$
   $5x - 2y = -6$

2. $y = 2x + 5$
   $y = -7 + 2x$

3. $2x + y = 4$
   $2y = 8 - 4x$

4. A system of consistent linear equations in two variables has ______ solution(s).

   (a) 0
   (b) 1
   (c) 2
   (d) at least 1

5. A system of inconsistent linear equations in two variables has ______ solution(s).

   (a) 0
   (b) 1
   (c) 2
   (d) at least 1

6. Graph the system of linear inequalities and indicate the solution by an approved method.
   $y \geq -3x - 1$
   $y < \frac{1}{2}x + 3$

7. In a linear programming problem, an inequality that models a limitation is called a(n)

   (a) solution
   (b) objective function
   (c) constraint
   (d) feasible region

8. The ______ contains the possible solutions to a linear programming problem.

   (a) optimal solution
   (b) feasible region
   (c) constraint

Sample 4e. A Sample Article

Abstract. This article illustrates many features of a mathematics document created with *Scientific Notebook*.

Mathematics and Text

Let H be a Hilbert space, C be a closed bounded convex subset of H, T a nonexpansive self map of C. Suppose that as $n \to \infty$, $a_{n,k} \to 0$ for each k, and $\gamma_n = \sum_{k=0}^{\infty}(a_{n,k+1} - a_{n,k})^+ \to 0$. Then for each x in C, $A_n x = \sum_{k=0}^{\infty} a_{n,k} T^k x$ converges weakly to a fixed point of T.

In-line and Displayed Mathematics

The equation

$$u_{tt} - \Delta u + u^5 + u|u|^{p-2} = 0 \text{ in } \mathbf{R}^3 \times [0,\infty[\qquad (1)$$

is numbered and it also would have the label "wave". You can use this label to jump to this equation using hypertext links.

Multi-Line Displays

Scientific Notebook provides a range of alignment options for multiline mathematical displays. Here is a series of multiline displays.

$$x = 17y \qquad (2)$$

$$\begin{aligned} y > a + b + c + d + e + f + g + h + i + j + \\ k + l + m + n + o + p \end{aligned} \qquad (3)$$

$$\begin{aligned} x &\ll y_1 + \cdots + y_n \\ &\leq z \end{aligned}$$

If $f(x) = x + 1$, then we will have

$$f([x+1]/[x+2]) = \{\, [x+1]/[x+2] \,\} + 1 = (2x+3)/(x+2)$$

$$\begin{aligned} 1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 \\ + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 = 190 \end{aligned}$$

Sample 5. Computer

This sample is transcribed using *Code for Computer Braille Notation 1987* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

Comment lines in programs begin with apostrophes. Those which are printed at the end of a line of code are brailled on the following line in cell 5. Those which are printed at the beginning of a line are brailled as printed.

A right-pointing arrow is used as a print line-continuation indicator to show that a line of code continues from one line to the next. This is omitted in braille.

Items in italic indicate placeholders for information supplied by the reader.

Items inside square brackets are optional.

Sans serif font is used in print for presenting sample code.

Tip Using an update query to change job titles is more efficient.

For more information about bookmarks, search Help for "Bookmark."

Access Basic sets the NoMatch property to **True** whenever a Find operation fails and the current record position is undefined. There may be a current record, but you have no way to tell which record it may be. If you want to be able to return to the previous current record following a failed Find operation, use a bookmark.

NoMatch is **False** whenever the operation succeeds. In this case, the current record position is the record found by one of the Find methods.

The following example illustrates how you can use the **FindNext** method to find all records that contain **Null** in the Phone field in a table named Customers.

```
Dim MyDb As Database, MySet As Recordset, Criterion As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Customers",DB_OPEN_DYNASET)

Criterion = "[Phone] Is Null"

MySet.FindFirst Criterion
Do Until MySet.NoMatch
    MsgBox MySet![Company Name] & " is missing a phone number."
    MySet.FindNext Criterion
Loop
```

Tip You can modify this example to include a call to the **InputBox** function instead of a call to the **MsgBox** function. This lets you enter a phone number for the found records. Then, you can use the **Edit** and **Update** methods to add the new phone numbers to the records.

► Using the Seek Method

For more information about setting the current index, search Help for "current index" or see "Sorting a Recordset" earlier in this chapter.

You use the **Seek** method to locate a record in a table-type **Recordset**. To locate a record in a dynaset or snapshot, use one of the Find methods described in the preceding section. When you use **Seek** to locate a record, Access Basic uses the table's current index, as defined by the **Index** property.

The syntax for the **Seek** method is:

table.**Seek** *comparison*, *key1*, *key2* ...

The *table* argument is a recordset variable that refers to the table you are searching through. The **Seek** method accepts a number of arguments, the first of which is *comparison*, a string that determines the kind of comparison that's performed. The following table lists the comparison strings you can use with **Seek**.

Comparison string	Description
"="	Equal to the specified key values
">="	Greater than or equal to the specified key values
">"	Greater than the specified key values
"<="	Less than or equal to the specified key values
"<"	Less than the specified key values

The *keyn* arguments are a series of one or more values that Access Basic compares to values in the table records. The following example opens a table named Products, and uses the **Seek** method to locate the first record containing a value of 1 in the Supplier ID field (which is a nonunique index field). It changes 1 to 2 and saves the change with the **Update** method. Subsequent passes through the loop locate the next record that satisfies the condition.

```
Dim MyDb As Database, MyTable As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
' Open Table.
Set MyTable = MyDb.OpenRecordset("Products", DB_OPEN_TABLE)

MyTable.Index = "Supplier ID"               ' Define current index.
MyTable.Seek "=", 1                         ' Seek record.

Do Until MyTable.NoMatch                    ' Until no record is found.
    MyTable.Edit                            ' Enable editing.
    MyTable("Supplier ID") = 2              ' Change Supplier ID.
    MyTable.Update                          ' Save changes.
    MyTable.Seek "=", 1                     ' Seek next record.
Loop                                        ' End of loop.

MyTable.Close                               ' Close Table.
```

If you use the **Seek** method on a **Recordset** object of type table without first setting the current index, a run-time error occurs.

The next example illustrates how you can create a function that uses the **Seek** method to locate a record using a multiple-field index.

```
Function GetFirstPrice(ByVal OrderID As Long, ByVal ProductID As Long)
➥ As Variant
```

```
Dim MyDb As Database, MyTable As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyTable = MyDb.OpenRecordset("Order Details", DB_OPEN_TABLE)

MyTable.Index = "PrimaryKey"
MyTable.Seek "=", OrderID, ProductID

If MyTable.NoMatch Then
    GetFirstPrice = Null
    MsgBox "Couldn't find order detail record."
Else
    GetFirstPrice = MyTable![Unit Price]
End If

MyTable.Close
MyDb.Close

End Function
```

For more information about using the **Seek** method, search Help for "Seek."

In this example, the table's primary key consists of two fields, Order ID and Product ID. When you call the GetFirstPrice function with a valid (existing) combination of Order ID and Product ID field values, the function returns the unit price from the found record. If the desired combination of field values can't be found in the table, the function returns **Null**.

If the current index is a multiple-field index, trailing key value can be omitted and are treated as **Null**. That is, you can leave off any number of key values from the end of a **Seek** method's *key* argument, but not from the beginning or middle.

Using Bookmarks Instead of Record Numbers

If you have used another database or programming environment, you may be accustomed to referring to record numbers. For example, you may have written code that opens a text file and thereafter refers to specific records by their relative position in the file. The first record in the file would be record 1, the second would be record 2, and so on.

In Microsoft Access, your view of records is usually a subset of the records in an entire table. Because the actual number of records can change at any time, especially in a multiuser environment, there's no absolute record number you can use to refer to a particular record. Instead, you can use bookmarks to identify and then return to a particular record.

A bookmark is a system-generated, randomly created string that uniquely identifies each record. To save a bookmark, you assign the value of the Bookmark property to a string variable. To return to the record, you set the Bookmark property to the value of the variable.

The following example illustrates how you can use a bookmark to save the current record position and then quickly return to that record position if a find or seek operation fails. If the operation fails, the current record position is undefined.

```
Dim MyDb As Database, MyTable As Recordset, MyBookmark As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyTable = MyDb.OpenRecordset("Customers", DB_OPEN_TABLE)

MyTable.Index = "Company Name"
...
MyBookmark = MyTable.Bookmark
MyTable.Seek ">=", "Z"

If MyTable.NoMatch Then
    MsgBox "Can't find a company name starting with 'Z'."
    MyTable.Bookmark = MyBookmark
End If

MyTable.Close
MyDb.Close
```

In this example, saving the Bookmark property, and then resetting the Bookmark property to its previous value when the **Seek** method fails, causes the previously current record to be the current record again.

You can also use the Bookmark property for the dynaset used by a form. This lets your code mark which record is displayed on the form, and to change the record being displayed, by setting the form's Bookmark property. For example, you can add to a form containing employee information a button that a user can click to show the record for an employee's supervisor. The following example illustrates the function you can call from the button's Click event.

```
Function ShowSupervisor() As Integer
    Dim MyForm As Form, MySet As Recordset, MyBookmark As String

    Set MyForm = Screen.ActiveForm
    Set MySet = MyForm.Recordset.Clone
    MySet.Bookmark = MyForm.Bookmark

    MySet.FindFirst "[Employee ID] = " & MySet![Supervisor]
    If MySet.NoMatch Then
        MsgBox "Couldn't find employee's supervisor."
        MyForm.Bookmark = MyBookmark
    Else
        MyForm.Bookmark = MySet.Bookmark
    End If
```

```
    MySet.Close
    MyDb.Close

End Function
```

Some attached tables, for example, Paradox® tables that have no primary key, may not support bookmarks. Therefore, you can neither use bookmarks on such tables, nor use bookmarks on **Recordset** objects or queries based on those tables. However, all snapshots support bookmarks, regardless of their underlying tables. You can determine whether a table supports bookmarks by checking the value of the Bookmarkable property, as in the following example.

```
If MySet.Bookmarkable Then
    MsgBox "The underlying table supports bookmarks."
Else
    MsgBox "The underlying table doesn't support bookmarks."
End If
```

If you try to use bookmarks on a **Recordset** that doesn't support bookmarks, a run-time error occurs.

When you close a **Recordset**, any bookmarks you saved become invalid. You can't use a bookmark obtained from one **Recordset** in another **Recordset**, even if both **Recordset** objects are based on the same underlying table or query. A run-time error occurs if you try to share bookmarks between **Recordset** objects. However, you can use a bookmark on the duplicate (clone) of a **Recordset** as the following example shows.

```
Dim MyDb As Database
Dim OrigRecordset As Recordset, DupRecordset As Recordset
Dim Placeholder As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
' Create first Recordset.
Set OrigRecordset = MyDb.OpenRecordset("Orders",DB_OPEN_DYNASET)

Placeholder = OrigRecordset.Bookmark     ' Save current record position.
Set DupRecordset = OrigRecordset.Clone() ' Create duplicate Recordset.
DupRecordset.Bookmark = Placeholder      ' Go to same record.
```

Changing Records

Once you've created a recordset variable of table or dynaset type, you can change, delete, or add new records. You can't change, delete, or add records to a snapshot-type **Recordset** object. This section presents the methods and procedures for changing data in table- and dynaset-type **Recordset** objects.

Modifying an Existing Record

Changing an existing record in a recordset variable is a four-step process:

- First, go to the desired record.
- Second, use the **Edit** method to prepare the current record for editing.
- Third, make the necessary changes to the record.
- Fourth, use the **Update** method to save the changes to the current record.

The following example illustrates how to change the job titles for all sales representatives in a table named Employees.

```
Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Employees")

MySet.MoveFirst
Do Until MySet.EOF
    If MySet![Title] = "Sales Representative" Then
        MySet.Edit
        MySet![Title] = "Account Executive"
        MySet.Update
    End If
    MySet.MoveNext
Loop

MySet.Close
MyDb.Close
```

If you don't use the **Edit** method before you try to change a value in the current record, a run-time error occurs.

Important If you change the current record, and then move to another record or close the **Recordset** without first using the **Update** method, your changes are lost without warning. For example, omitting the **Update** method from the preceding example results in no changes being made to the Employees table.

Dynaset-type **Recordset** objects can be based on a multiple-table query, with the query often implementing a one-to-many relationship. For example, suppose you want to create a multiple-table query that combines fields from the Orders and Order Details tables. Normally, you are prevented from changing values in the Orders table because it's on the "one" side of the relationship. Depending on your application, however, you may want to allow changes to the Orders table. To allow changes to the "one" side of a one-to-many relationship, you can use the **OpenRecordset** method's DB_INCONSISTENT constant to create an *inconsistent* dynaset. This allows you to freely change values on the "one" side of the relationship.

```
Set MySet = MyDb.OpenRecordset("Sales Totals", DB_INCONSISTENT)
```

Important When you update an inconsistent dynaset, you can easily destroy the relational integrity of the data in the dynaset. You must take care to understand how the data is related across the one-to-many relationship and update the values on both sides in a way that preserves data integrity.

The DB_INCONSISTENT option is available only for dynaset-type **Recordset** objects. It's ignored for table- and snapshot-types.

Even with an inconsistent **Recordset**, some fields may not be updatable. For example, you can't change the value of a Counter field, and a **Recordset** based on certain attached tables may not be updatable.

Deleting an Existing Record

For more details on the **Delete** method, search Help for "Delete."

You can delete an existing record in a table- or dynaset-type **Recordset** using the **Delete** method. You can't delete records from a snapshot-type **Recordset**. The following example deletes all the records for trainees in a table named Employees.

```
Dim MyDb As Database, MySet As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Employees")

MySet.MoveFirst
Do Until MySet.EOF
    If MySet![Title] = "Trainee" Then
        MySet.Delete
    End If
    MySet.MoveNext
Loop

MySet.Close
MyDb.Close
```

When you use the **Delete** method, Microsoft Access immediately deletes the current record, without any warning or prompting. Deleting a record doesn't automatically cause the next record to become the current record; to move to the next record you must use the **MoveNext** method.

Adding a New Record

Adding a new record to a dynaset- or table-type **Recordset** is a three-step process:

- First, use the **AddNew** method to prepare a new record for editing.
- Second, assign values to each of the record's fields.
- Third, use the **Update** method to save the new record.

The following example adds a new record to a table named Shippers.

```
Dim MyDb As Database, MySet As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Shippers")

MySet.AddNew
MySet![Company Name] = "Global Parcel Service"
...                                         ' Set remaining fields.
MySet.Update

MySet.Close
MyDb.Close
```

For additional information on the **AddNew** method, search Help for "AddNew."

When you use the **AddNew** method, Microsoft Access prepares a new, blank record and makes it the current record. When you use the **Update** method to save the new record, the record that was current before you used **AddNew** becomes the current record again. The new record's position in the **Recordset** depends on whether you added the record to a dynaset- or a table-type **Recordset.**

If you add a record to a dynaset-type **Recordset,** the new record appears at the end of the **Recordset,** no matter how the **Recordset** is sorted. To force the new record to appear in its properly sorted position, either recreate the **Recordset** or create a second **Recordset** based on the original.

If you add a record to a table-type **Recordset,** the record appears positioned according to the current index, or at the end of the table if there is no primary key and no current index.

Important If you use the **AddNew** method to add a new record, and then move to another record or close the **Recordset** without first using the **Update** method, your changes are lost without warning. For example, omitting the **Update** method from the preceding example results in no changes being made to the Shippers table.

Making Bulk Changes

Many of the changes you might otherwise perform in a loop can be done more efficiently with an update or delete query. For example, the task of changing all the job titles for sales representatives, illustrated in several previous examples by looping through each employee's record and inspecting the existing job title, can be quickly accomplished by an update query.

```
Dim MyDb As Database, MyQuery As QueryDef

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyQuery = MyDb.CreateQueryDef()
MyQuery.Name = "Change Job Titles"                    ' Create query.
MyQuery.SQL = "UPDATE DISTINCTROW Employees           ' Set SQL property.
➥ SET Employees!Title = 'Account Executive' WHERE Employees!Title =
➥ 'Sales Representative';"
MyDb.QueryDefs.Append MyQuery

MyQuery.Execute                               ' Invoke query.
MyQuery.Close                                 ' Close query.
MyDb.QueryDefs.Delete MyQuery.Name            ' Delete query.
```

Depending on your task, you may also find a delete query is more efficient than code that loops through records looking for records to delete.

Using Transactions to Control Changes

Often, to enforce data integrity, a set of operations must be considered as a single unit. For example, the transfer of funds from one bank account to another consists of two operations: entering a debit in one account and a matching credit in the other account. In practice, either both operations must succeed or neither operation should be processed.

In this situation, the debit and credit operations are a single unit, or a *transaction*. In Microsoft Access, a transaction is a set of operations that are *committed* (saved) if and only if all of the operations succeed. If any of the operations fails, all of the operations that succeeded are *rolled back* (canceled), and the data is returned to the state it was in before the transaction began.

Access Basic supports three transaction methods: one for starting, one for committing, and one for rolling back a transaction:

- The **BeginTrans** method begins a new transaction.
- The **CommitTrans** method commits all changes made to data since the most recent **BeginTrans** method.

- The **Rollback** method rolls back, or cancels, all changes made to data since the most recent **BeginTrans** method.

BeginTrans, **CommitTrans**, and **Rollback** are all methods of the **Workspace** object.

The following example changes the job title of all sales representatives in the Employees table of the current database. After the **BeginTrans** method starts a transaction that isolates all the changes made to the Employees table, the **CommitTrans** method saves the changes. Notice that you can use the **Rollback** method to undo changes that you saved with the **Update** method.

```
Function ChangeTitle
    Const MB_ICONQUESTION = 32
    Const YES = 6
    Const YES_NO = 4

    Dim CRLF As String, EmployeeName As String
    Dim Message As String, Prompt As String
    Dim MyDb As Database, MyWorkSpace as Workspace, MyTable As Recordset

    CRLF = Chr$(13) & Chr$(10)
    Prompt = "Change title to Account Executive?"

    ' Get current database.
    Set MyWorkspace = DBEngine.Workspaces(0)
    Set MyDb = MyWorkspace.Databases(0)
    ' Open table.
    Set MyTable = MyDb.OpenRecordset("Employees", DB_OPEN_TABLE)

    MyWorkspace.BeginTrans          ' Start of transaction scope.

    Do Until MyTable.EOF

        If MyTable![Title] = "Sales Representative" Then
            EmployeeName = MyTable![Last Name] & ", " & MyTable![First
            ➥ Name]
            Message = "Employee: " & EmployeeName & CRLF & CRLF
            If MsgBox(Message & Prompt, MB_ICONQUESTION & YES_NO,
            ➥ "Change Job Title") = YES Then

                MyTable.Edit                                ' Enable
                                                            ' editing.
                MyTable![Title] = "Account Executive"       ' Change title.
                MyTable.Update                              ' Save changes.
            End If
        End If
```

```
        MyTable.MoveNext                           ' Move to next record.
    Loop

    If MsgBox("Save all changes?", MB_ICONQUESTION + YES_NO, "Save
    ➥ Changes") = YES Then
        MyWorkspace.CommitTrans                    ' Commit changes.
    Else
        MyWorkspace.Rollback                       ' Undo changes.
    End If

    MyTable.Close                                  ' Close table.

End Function
```

Transactions and Performance

In addition to ensuring data integrity, transactions are important for improving database performance in many situations. Transactions provide speed increases because they tell the Jet database engine when it can flush memory caches to disk. Transactions bracket related operations so they can be cached in memory until either a **CommitTrans** or **Rollback** method occurs. Storing a set of operations in memory is much faster than writing every operation to disk, as occurs without transactions.

For more information on writing applications for use in a multiuser environment, see Chapter 12, "Developing Multiuser Applications."

Transactions are also a good way to increase *concurrency*—the number of users that can access a database simultaneously—in multiuser applications. For example, the following sequence allows more users to get at data simultaneously without locking conflicts:

```
MyWorkspace.BeginTrans
MyRs = MyDb.OpenRecordset("Long running query")
MyWorkspace.CommitTrans

Debug.Print MyRs!Field1
...
```

Computer Sample in CBC

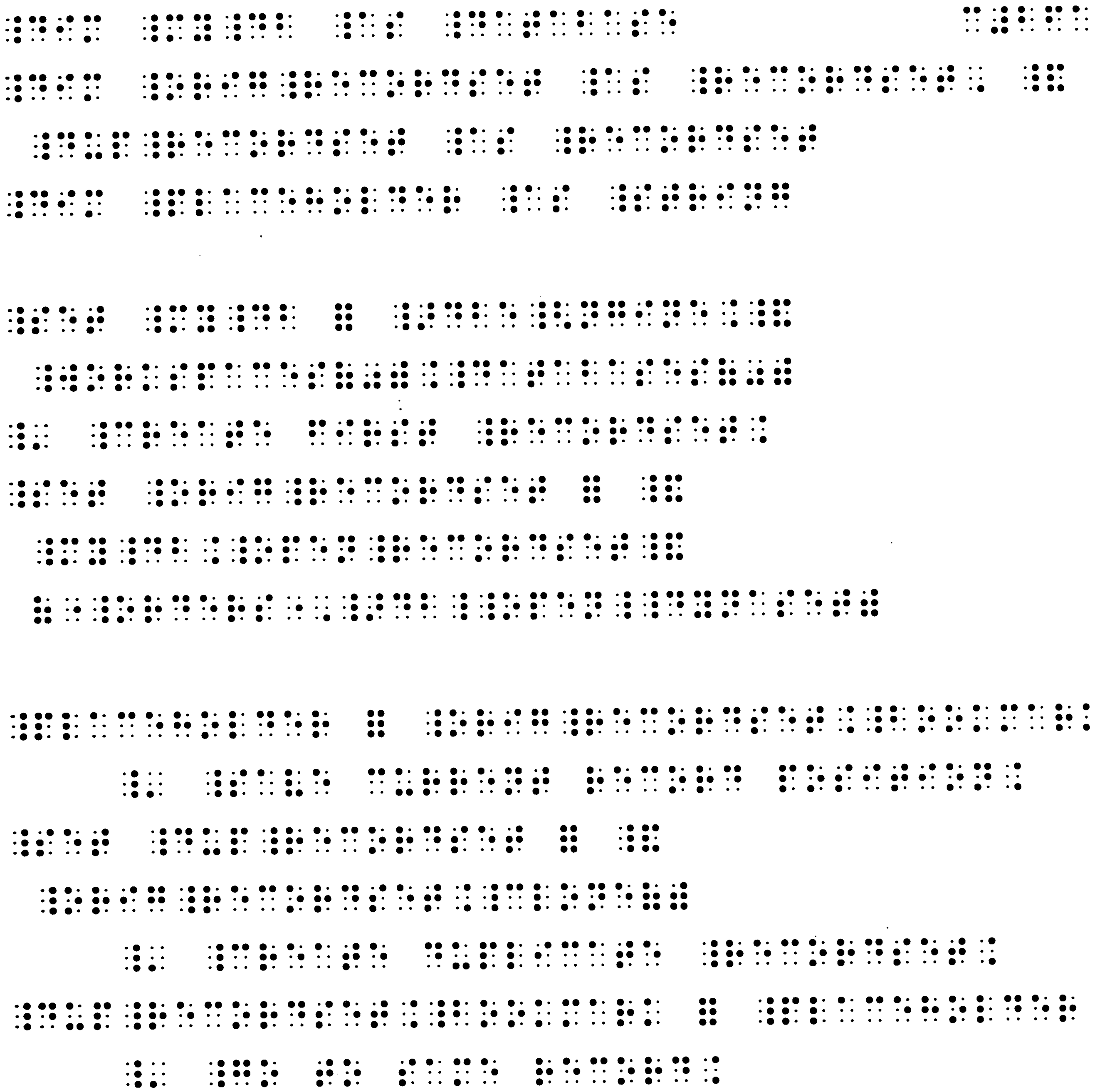
CBC Symbols

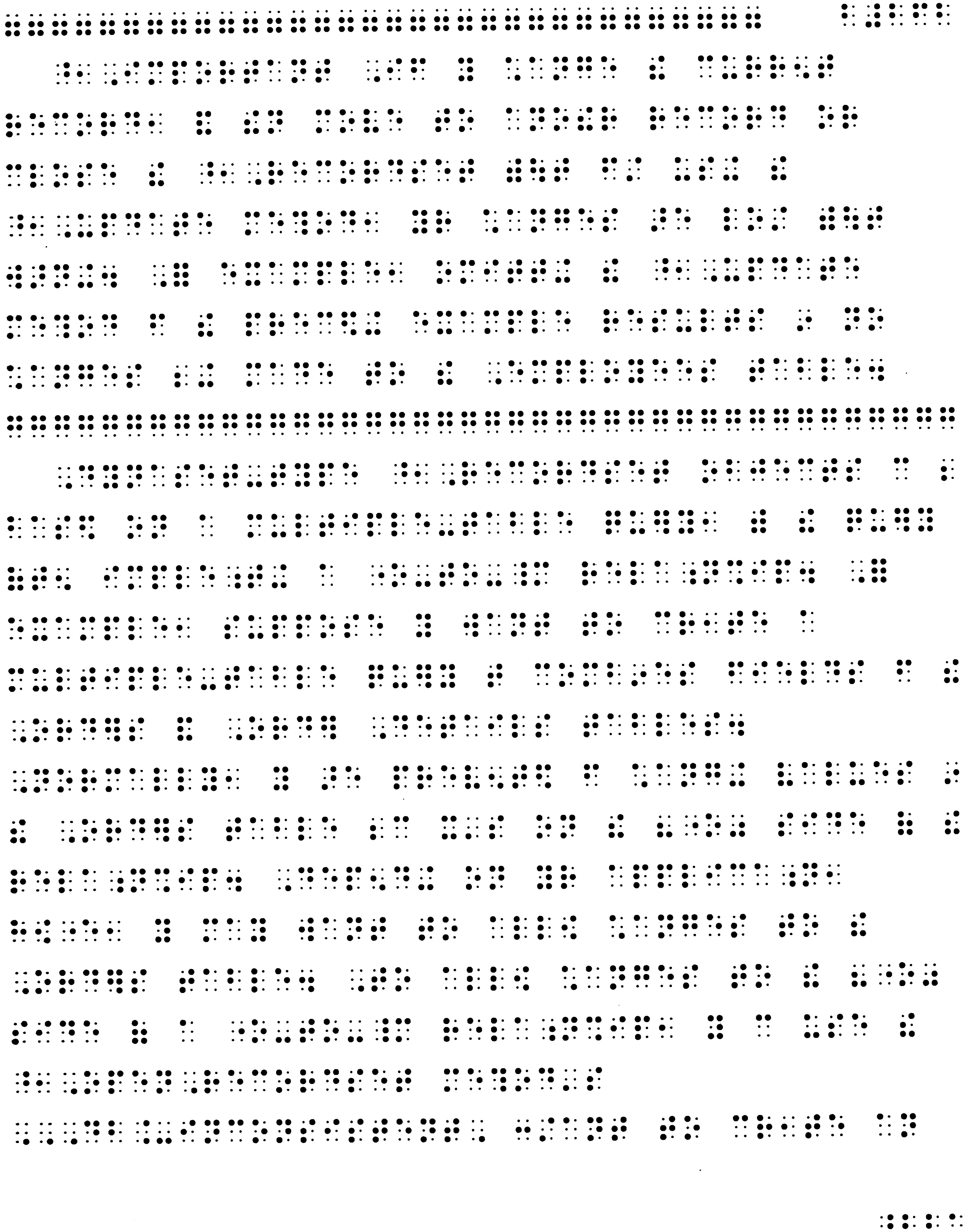
begin Computer Braille Code	double quote
end Computer Braille Code	single quote, apostrophe
shift indicator	equal sign
caps lock indicator	greater than sign
caps release indicator	less than sign
begin emphasis	plus sign
end emphasis	opening parenthesis
transcriber's option symbol: begin italic type within the Computer Braille Code	closing parenthesis
termination indicator: end italic type within the Computer Braille Code	opening bracket
continuation indicator	closing bracket
isolated lower-cell indicator	underscore
transcriber's note symbol	ampersand
braille reference indicator; used to mark a marginal note	exclamation point
period	semicolon
ellipsis	dollar sign
comma	question mark
	colon

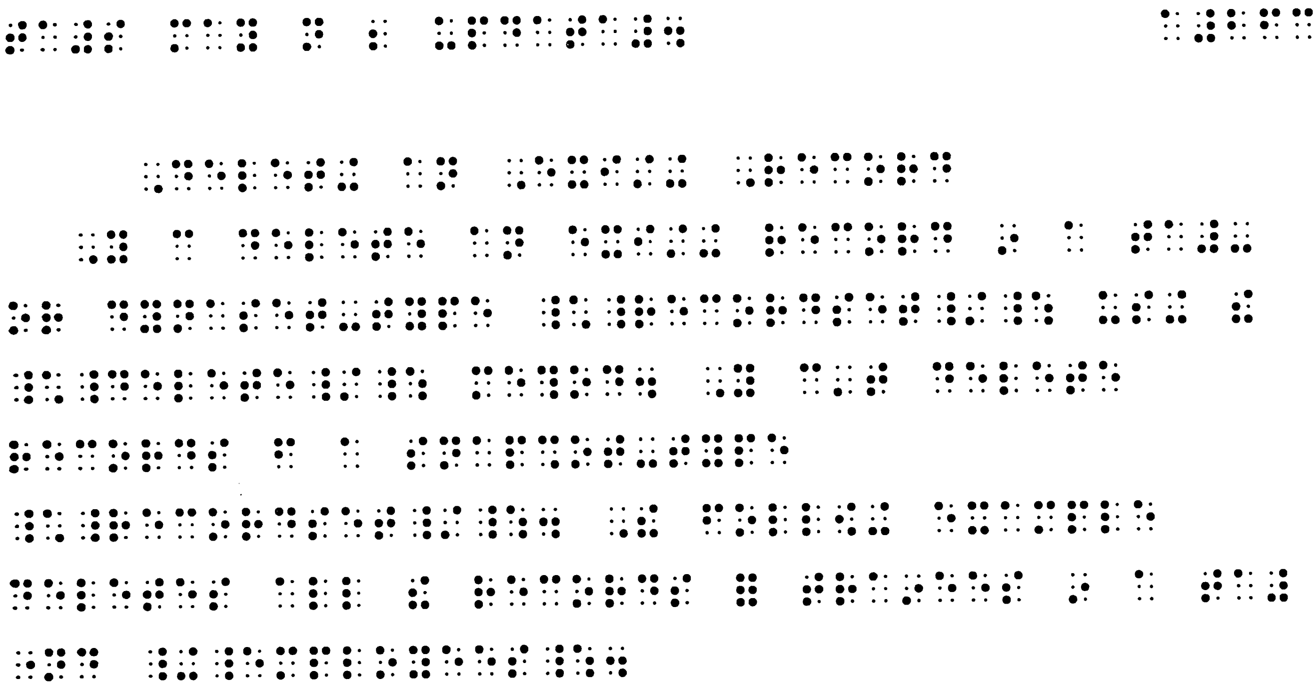
Computer Sample in UEBC

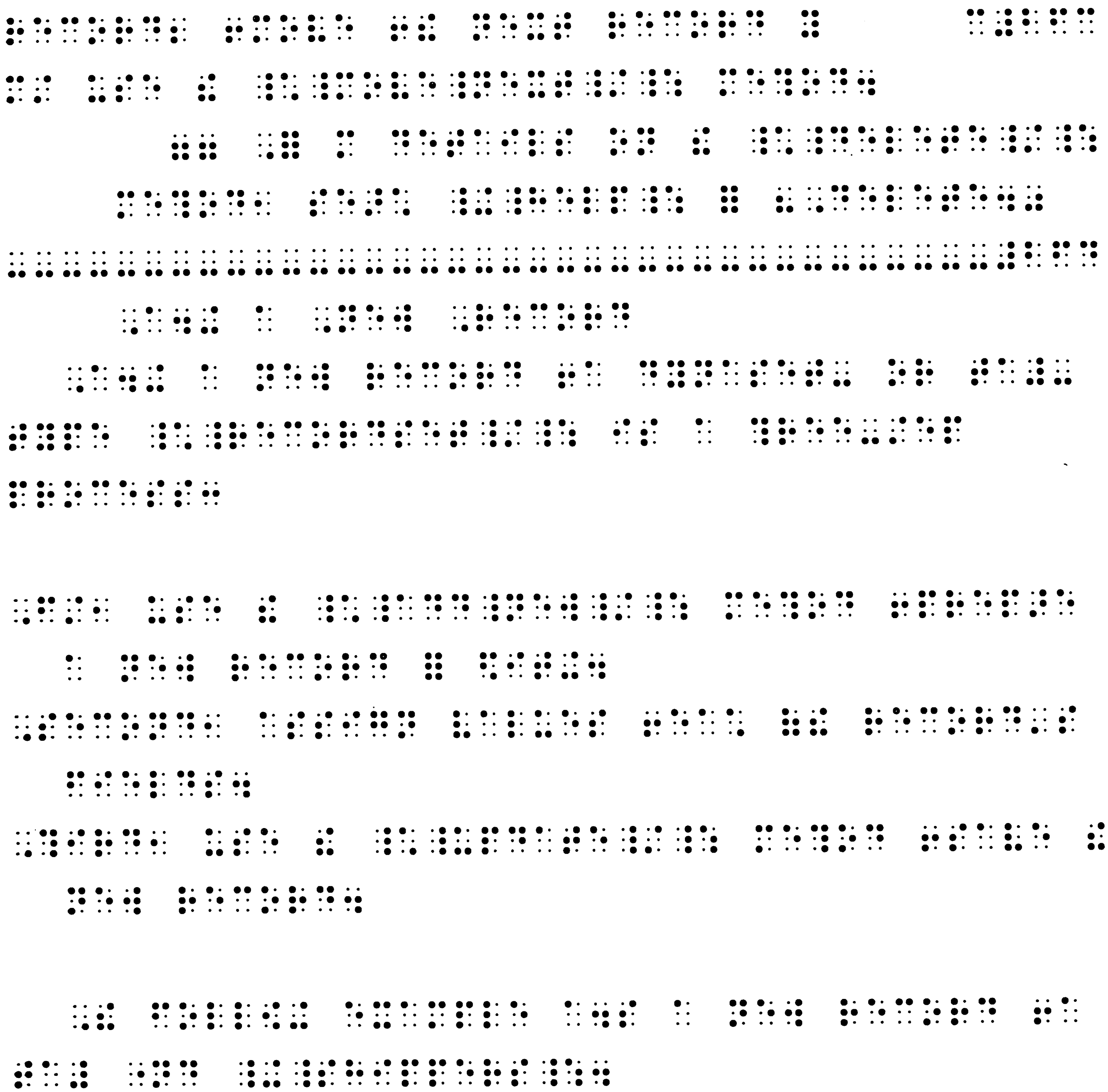
UEBC Symbols

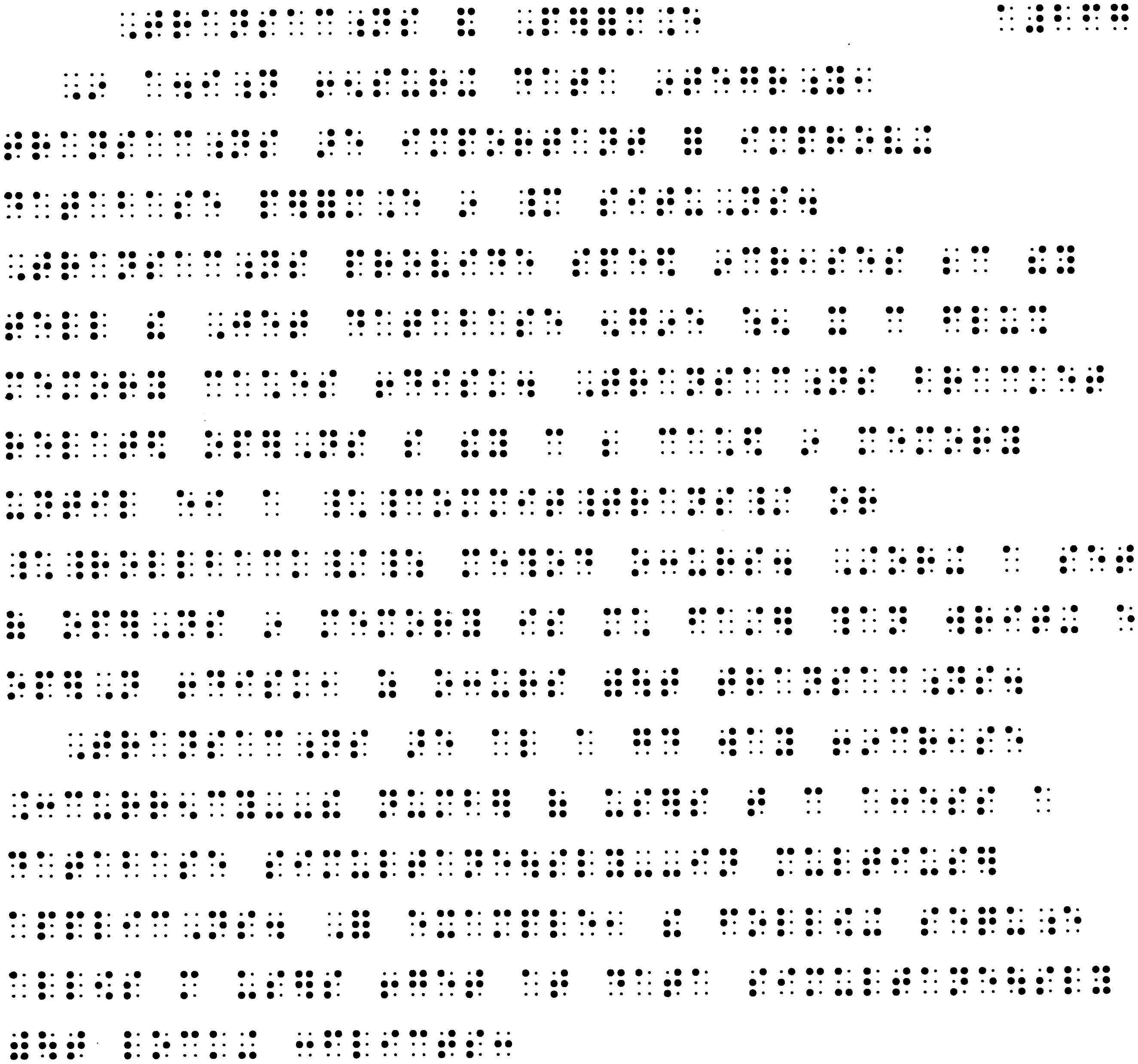
⠘⠂	bold word
⠘⠶	begin bold passage
⠘	(before a space) end bold passage
⠨⠂	italic word
⠨⠶	begin italic passage
⠨⠄	end italic
⠨	(before a space) end italic passage
⠐	(at end of line) continuation indicator
⠐ ⠐	(at end of line) continuation indicator with a space
⠰⠰⠰	begin grade one passage
⠰	(before a space) end grade one passage
⠠⠠⠠	begin capitalized passage
⠠⠄	end capitals within a sequence
⠠	(before a space) end capitalized passage
⠲⠲⠲	ellipsis
⠐⠶	equal sign
⠐⠖	plus sign
⠈⠜	greater than sign
⠈⠣	less than sign
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠨⠣	opening bracket
⠨⠜	closing bracket
⠠⠶	nondirectional double quote
⠄	nondirectional single quote, apostrophe
⠨⠤	underscore
⠈⠯	ampersand
⠈⠨⠗	registered symbol (circled R)
⠸⠲	bullet
⠈⠎	dollar sign
⠠⠤	dash











Sample 6. Chemistry

This sample is transcribed using *Braille Code for Chemical Notation 1997* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

In the UEBC sample, chain and ring diagrams are transcribed using grade one mode. Runover locators in these diagrams are preceded by an **r**.

B.3 CARBOHYDRATES: THE ENERGIZERS

Sugars, starch, and cellulose are all carbohydrates.

Carbohydrates are compounds composed of only three elements—carbon, hydrogen, and oxygen. For example, glucose, the key energy-releasing carbohydrate in biological systems, has the formula $C_6H_{12}O_6$. When such formulas were first discovered, chemists were tempted to write the glucose formula as $C_6(H_2O)_6$—implying a chemical combination of carbon and water. So, they invented the term "carbo-hydrate," or water-containing carbon compound. We now know that water molecules are not actually present in carbohydrates, but the name has persisted.

Every moment of your life, carbohydrates are oxidized in your body to produce energy, CO_2, and H_2O.

Carbohydrates may be simple sugars such as glucose or composed of two or more simple sugar molecules combined in various ways (Table 3). Simple sugars are called ***monosaccharides***. The most common monosaccharide molecules contain five or six carbon atoms bonded together. As shown in Figure 2, glucose (and most other monosaccharides) can exist either in a chain or a ring form. Do both forms have the same molecular formula? (Check by counting the atoms.)

Figure 2 Structural formulas for glucose. The chain and ring forms are interconvertible; the ring form predominates in the body.

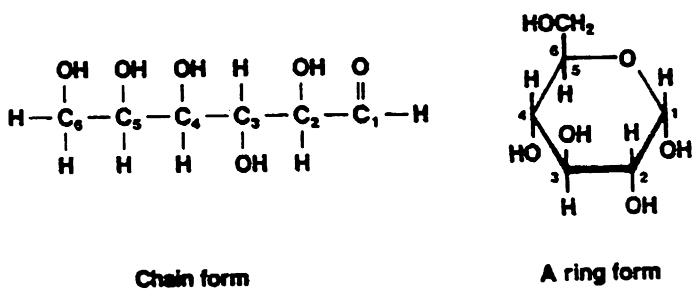
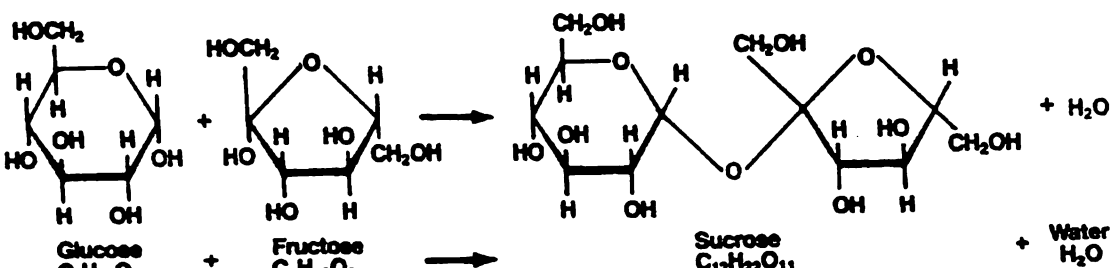


Figure 3 Formation of sucrose. The two shaded —OH groups react with the elimination of one H_2O molecule.



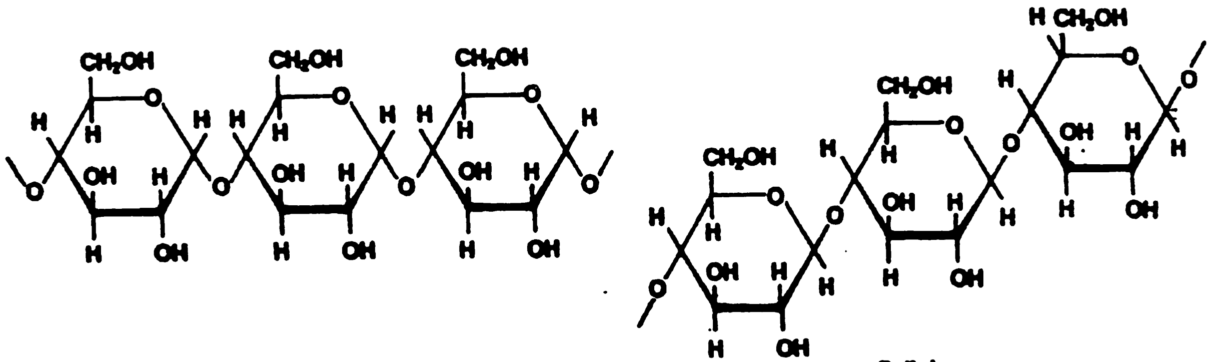


Figure 4 Polysaccharides. Starch and cellulose are both polymers of glucose. They differ in the arrangements of the bonds that join the glucose monomers.

Sugar molecules composed of two simple sugar units are called ***disaccharides.*** Sucrose ($C_{12}H_{22}O_{11}$—ordinary table sugar) is a disaccharide in which the ring forms of glucose and fructose are joined (see Figure 3). As the molecular structures suggest, monosaccharides and disaccharides are composed of polar molecules. Thus they tend to be highly soluble in water, a polar solvent.

Recall the "like dissolves like" rule.

Polymers composed of units of simple sugar molecules are called ***polysaccharides*** (Figure 4). Starch, a major component of grains and many vegetables, is a polysaccharide composed of glucose units. Cellulose, the fibrous or woody material of plants and trees, is another polysaccharide formed from glucose. The types of carbohydrates are summarized in Table 3.

Remember the discussion of polyethylene on page 201. A polymer is a large molecule composed of many smaller molecular units chemically bonded together.

Carbohydrates are all sugars or polymers of sugars.

Table 3 **CARBOHYDRATES**

Classification and Examples	Composition	Formula	Common Name or Source
Monosaccharides		$C_6H_{12}O_6$	
Glucose	—		Blood sugar
Fructose	—		Fruit sugar
Galactose	—		—
Disaccharides		$C_{12}H_{22}O_{11}$	
Sucrose	Fructose + glucose		Cane sugar
Lactose	Galactose + glucose		Milk sugar
Maltose	Glucose + glucose		Germinating seeds
Polysaccharides	Glucose polymers	—	
Starch			Plants
Glycogen			Animals
Cellulose			Fiber

During photosynthesis, green plants produce glucose. The overall reaction is as follows:

$$6\,CO_2 + 6\,H_2O + 686\text{ kcal} \rightarrow C_6H_{12}O_6 + 6\,O_2$$

Carbon dioxide	Water	Solar energy	Glucose	Oxygen gas

Plants build these glucose molecules either into starch for energy storage, or into cellulose, becoming part of the plant's structure.

There are exceptions—cows and termites, for example, can digest cellulose.

Sugars and starch are rapidly digested in your body, making them convenient sources of energy. Cellulose is not digested. The glucose units in cellulose are bonded together differently than they are in starch (see Figure 4). The slight difference in bonding makes cellulose indigestible by most animals. However, cellulose, sometimes called fiber, is needed in the diet to keep the digestive system functioning properly.

Sugars and starch are the major energy-delivering substances in our diets. Even the smallest muscle twitch or thought requires energy. The body obtains most of this energy by burning the glucose molecules in sugars and starch. Each gram of carbohydrate delivers about 4 Calories of energy.

1 g carbohydrate = 4 Cal energy

Nutritionists recommend that about 60% of food energy come from carbohydrates. Most of the world's population obtains carbohydrates by eating grains. These grains are often consumed as rice, corn meal, wheat tortillas, bread, and pasta. In the United States we tend to eat more wheat breads and potatoes for carbohydrates than people do elsewhere. In all countries, fruits and vegetables also provide carbohydrates. Meats provide a small amount of carbohydrate in the form of glycogen, which is how animals store glucose. On average, each U.S. citizen consumes more than 90 lb (40 kg) of table sugar each year in beverages, breads, and cakes and as a sweetener. A 12-ounce non-diet cola drink contains nine teaspoons of sugar.

B.4 FATS: STORED ENERGY WITH A BAD NAME

Unlike carbohydrate and protein, the word "fat" has acquired its own general (and somewhat negative) meaning. To most people, "You're too fat" means that the person looks overweight. However, from a chemist's point of view, ***fats*** are a major category of biomolecules which have their own special characteristics and functions, just as carbohydrates and proteins do.

Fats are a significant part of our diet. They're present in meat, fish, and poultry; salad dressings and oils; dairy products; and grains. When our bodies take in more food than is needed for energy, much of the excess is converted to fat molecules and stored in the body. If food intake is not large enough to supply the body's energy needs, the body begins to burn stored fat.

Like carbohydrates, fats are composed of carbon, hydrogen, and oxygen. However, fats contain less oxygen than carbohydrates and contain more stored energy. Gram-for-gram, fat contains over twice the energy found in carbohydrates. Fats are generally nonpolar in nature and are only sparingly soluble in water. Because of their solubility and energy-storing properties, fat molecules are more like hydrocarbons than carbohydrates.

Fats are members of the class of biomolecules called ***lipids.*** Some lipids are builder molecules that form cell membranes. Others become hormones—chemical messengers that regulate processes in the body.

A typical fat molecule is a combination of a simple three-carbon alcohol called ***glycerol*** with three fatty acid molecules. (The formation of a typical fat is shown in Figure 5.) ***Fatty acids*** are a class of compounds made up of a long hydrocarbon chain with a carboxyl group (—COOH) at one end. The reaction producing a fat molecule is similar to one you already completed in the laboratory (page 204)—the production of an ester, methyl

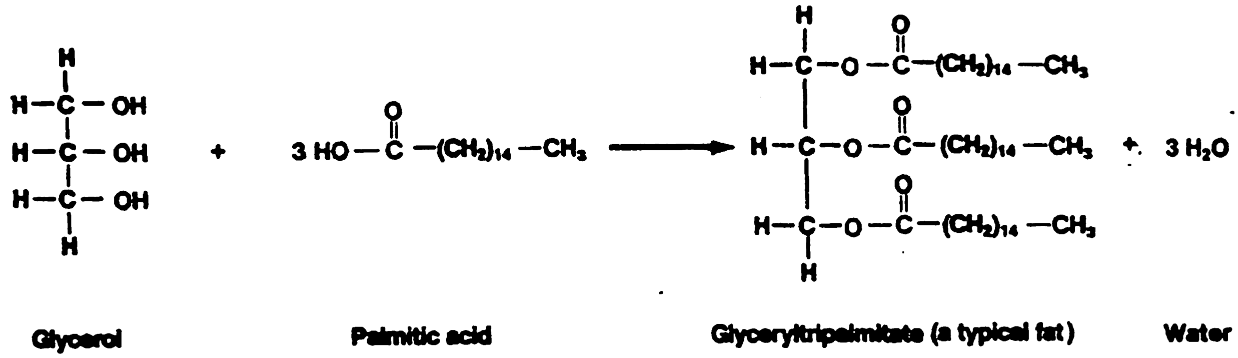


Figure 5 Formation of a typical fat, a triglyceride. Glycerol and three molecules of fatty acid combine in condensation reaction to form a triester and eliminate three water molecules.

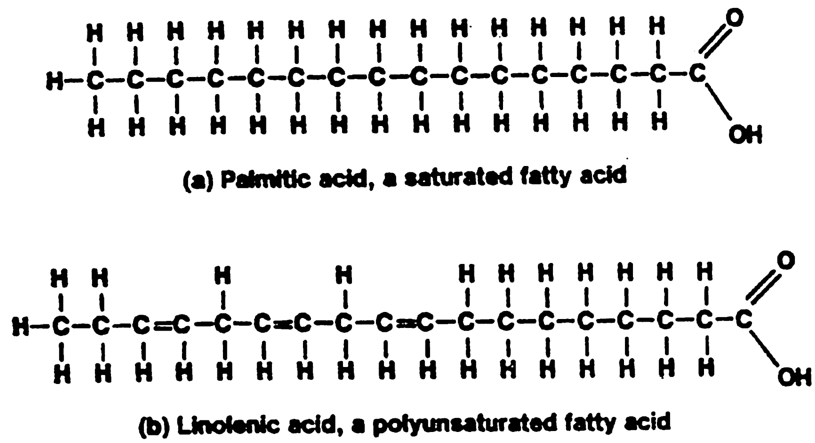


Figure 6 Typical fatty acids.

salicylate. However, here a molecule containing *three* —OH groups reacts with *three* molecules of acid. Three molecules of water are eliminated, producing a molecule containing three ester groups instead of one. Such a fat is known as a ***triglyceride.***

Recall that hydrocarbons may be saturated (when containing only single carbon-carbon bonds) or unsaturated (when containing double or triple carbon-carbon bonds). Likewise, hydrocarbon chains in fatty acids are either saturated (Figure 6a) or unsaturated (Figure 6b). Fats containing saturated fatty acids are called ***saturated fats;*** fats containing unsaturated fatty acids are called ***unsaturated fats.*** Because of differences in their bonds, saturated and unsaturated fats participate differently in body chemistry.

YOUR TURN

Functional Groups in Biomolecules

As you discovered in the Petroleum unit, functional groups strongly influence the properties of organic compounds.

Some classes of organic compounds in which particular functional groups appear are listed in the margin, with formulas written in their condensed forms (each R represents a hydrocarbon segment).

ROH
Alcohol

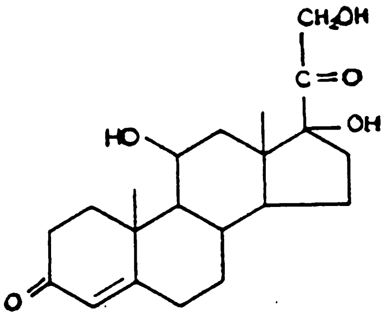
ROR
Ether

R—C(=O)—H
Aldehyde

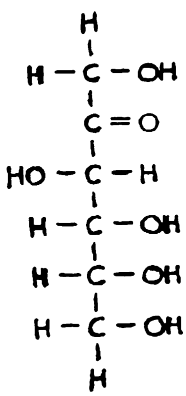
R—C(=O)—R
Ketone

R—C(=O)—O—H
Carboxylic acid

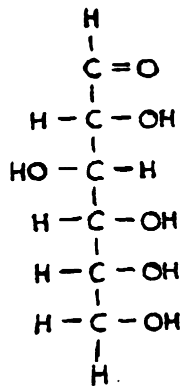
R—C(=O)—O—R
Ester



Cortisol



Fructose



Glucose

A molecule may contain more than one functional group. For example, look at the structure of cortisol (a lipid) in the margin. Cortisol is a hormone which, when released during starvation, makes it possible to use energy from protein. Note that cortisol contains several functional groups: three —OH groups, two C=O groups, and one C=C double bond.

1. Refer to the two forms of glucose shown in Figure 2.
   a. Draw both chain and ring structures for glucose.
   b. Circle and identify the functional group(s) found in the chain structure.
   c. Examine the numbering of the carbon atoms and the functional groups attached to each carbon atom.
      (1) Which functional groups apparently react to form the ring structure?
      (2) On which carbon atoms do these functional groups appear?
2. Compare the straight-chain structures of fructose and glucose (See margin.) Describe the differences in the structures of these two monosaccharides.
3. In general, alcohols react with organic (carboxylic) acids to form esters. Using the equation shown in Figure 5 as a guide, write an equation (including structures) for the reaction of stearic acid (a fatty acid) with glycerol to form glyceryl tristearate (a fat). Stearic acid has this structural formula:

$$HO-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_{16}CH_3$$

4. Copy the following molecular structure on your own paper:

$$HO-\overset{\overset{\displaystyle O}{\|}}{C}-(CH_2)_7-\overset{\overset{\displaystyle H}{|}}{C}=\overset{\overset{\displaystyle H}{|}}{C}-(CH_2)_7CH_3$$

   a. Circle and identify the functional group(s).
   b. Is this molecule a carbohydrate or a fatty acid? Why?
   c. Is it saturated or unsaturated? Why?
   d. Rewrite the molecular structure to show the carbon atoms in a continuous chain.

Saturated fats appear to contribute to coronary heart disease.

The term ***polyunsaturated,*** often used in food advertising, means that the food contains fats with two or more carbon-carbon double bonds in each fatty acid molecule. The term has become familiar because increasing evidence suggests that saturated fats may contribute to health problems, while some unsaturated fats may not. Saturated fats are associated with formation of plaque (fat-like or fibrous matter), which can block arteries. The result is a condition known as "hardening of the arteries," or atherosclerosis, a particular threat to coronary (heart) arteries and arteries leading to the brain. If coronary arteries are blocked, a heart attack can result, damaging the heart muscle.

Chemistry Sample in Chemistry Code

Nemeth Symbols

- baseline indicator
- decimal point
- general reference indicator denoting marginal material
- punctuation indicator
- opening parenthesis
- closing parenthesis
- begin subscript
- return to baseline after subscript
- mathematical comma
- plus sign
- right arrow
- transcriber's note symbol
- percent sign

Chemistry Code Symbols

- single horizontal bond
- bold horizontal bond
- double horizontal bond
- single vertical bond
- double vertical bond
- unlabeled vertex
- lower left to upper right oblique bond
- upper left to lower right oblique bond
- lower left to upper right, double, oblique bond
- up-pointing, filled in, oblique wedge

Chemistry Sample in UEBC

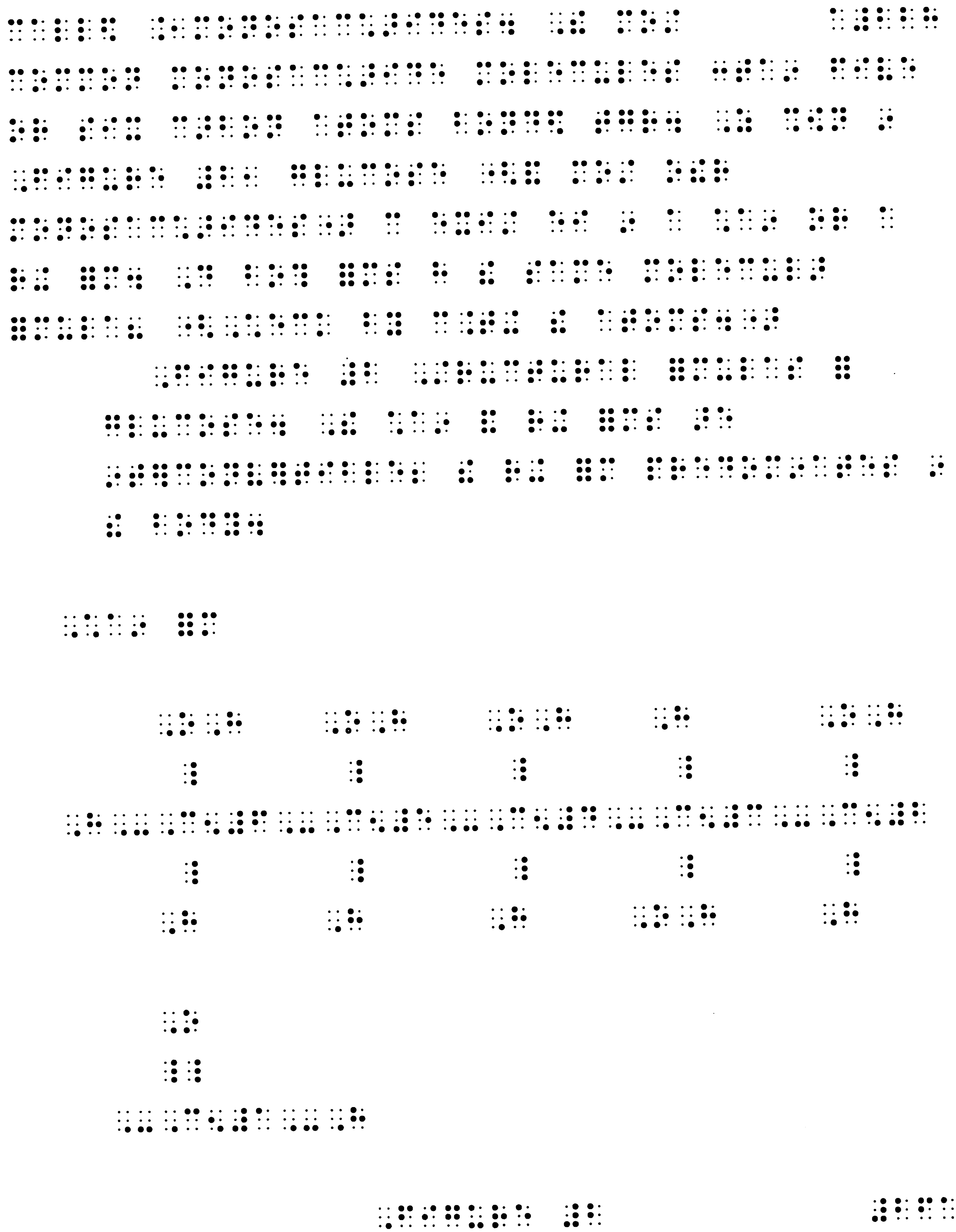
UEBC Symbols

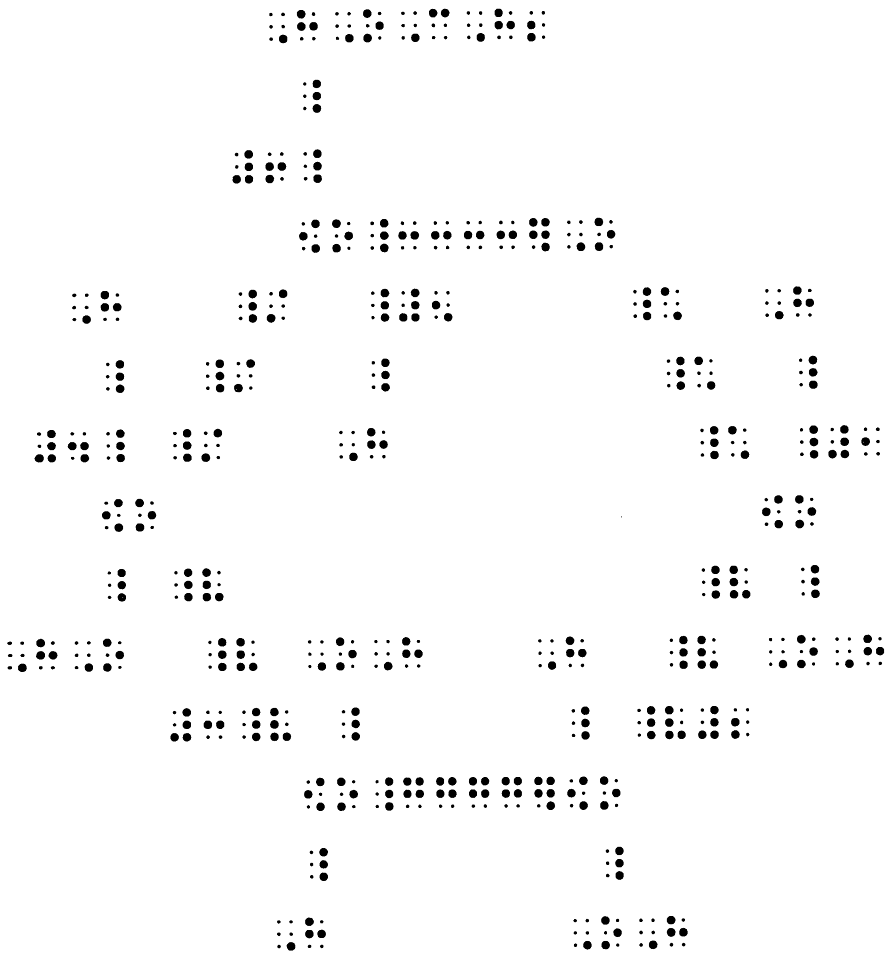
⠠⠠⠠	begin capitalized passage
⠠	(before a space) end capitalized passage
⠨⠂	italic word
⠨⠶	begin italic passage
⠨	(before a space) end italic passage
⠰	grade one symbol
⠰⠰	grade one word
⠰⠰⠰	begin grade one passage
⠰⠄	end grade one within a sequence
⠰	(before a space) end grade one passage
⠠⠤	dash
⠢	subscript next item
⠐⠣	opening parenthesis
⠐⠜	closing parenthesis
⠠⠤	single horizontal bond
⠨⠠⠤	double horizontal bond
⠐⠖	plus sign
⠐⠶	equal sign
⠳⠕	right arrow
⠠⠄	transcriber's note symbol
⠨⠴	percent sign

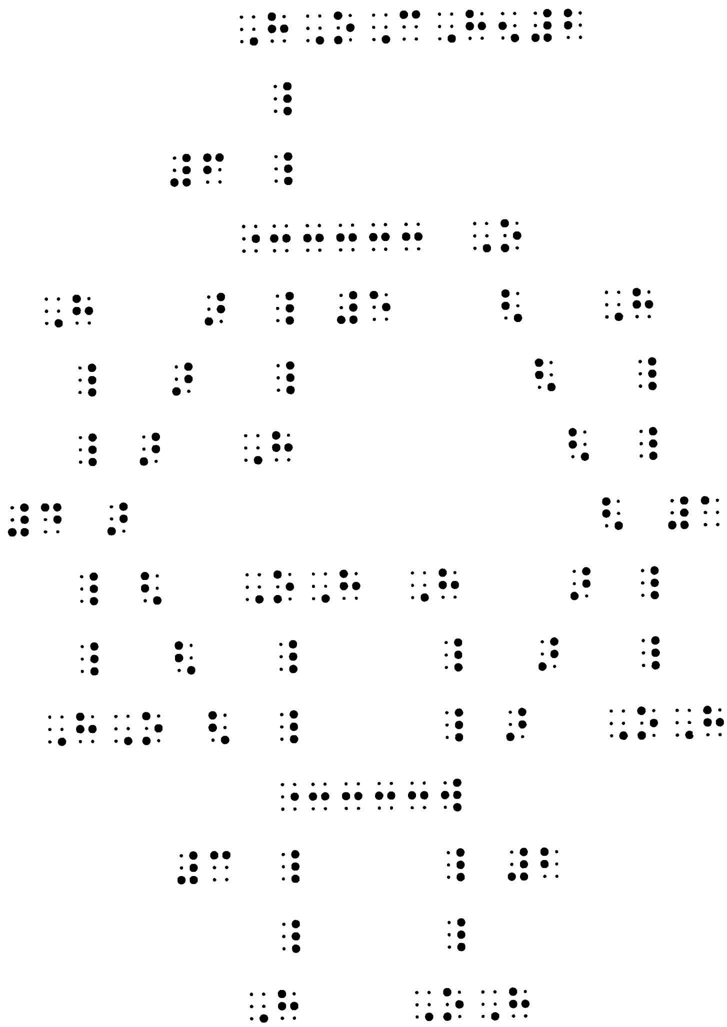
Line Symbols in UEBC

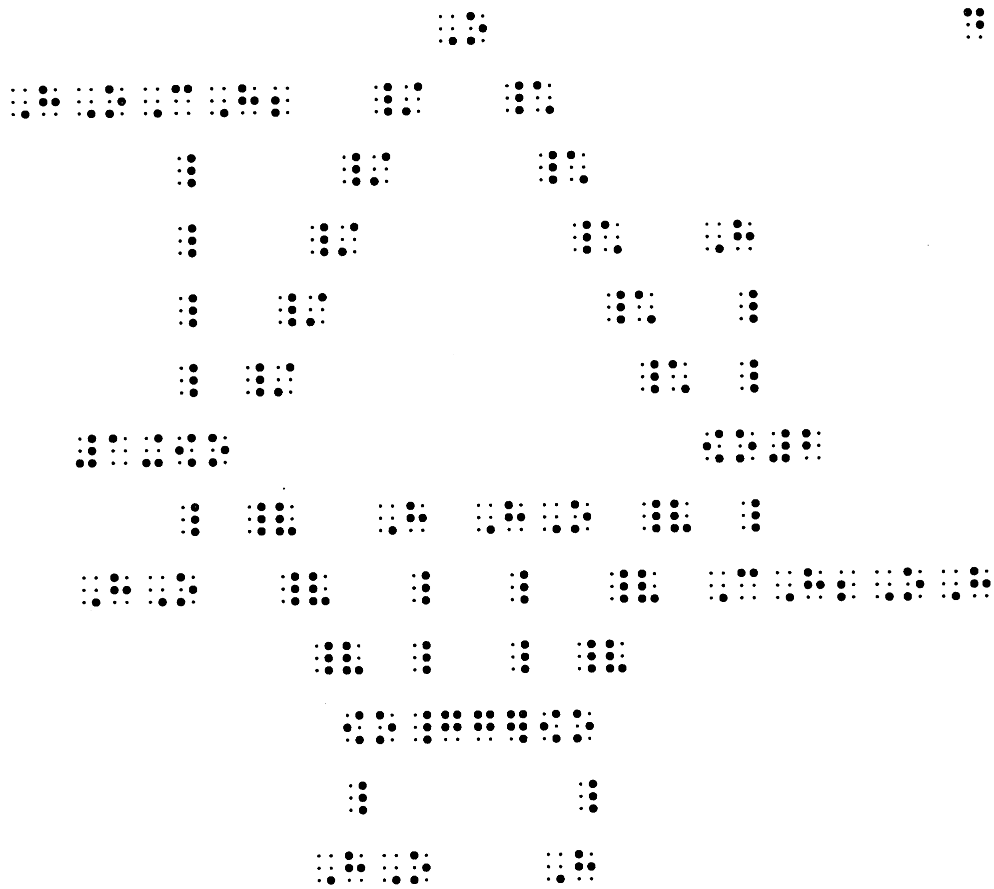
⠸	single vertical line
⠸⠸	double vertical line
⠐⠒⠒	single horizontal line
⠜	single right-leaning diagonal line
⠣	single left-leaning diagonal line
⠜⠜	double right-leaning diagonal line
⠺	crossing with vertical line
⠚	right corner with upward vertical

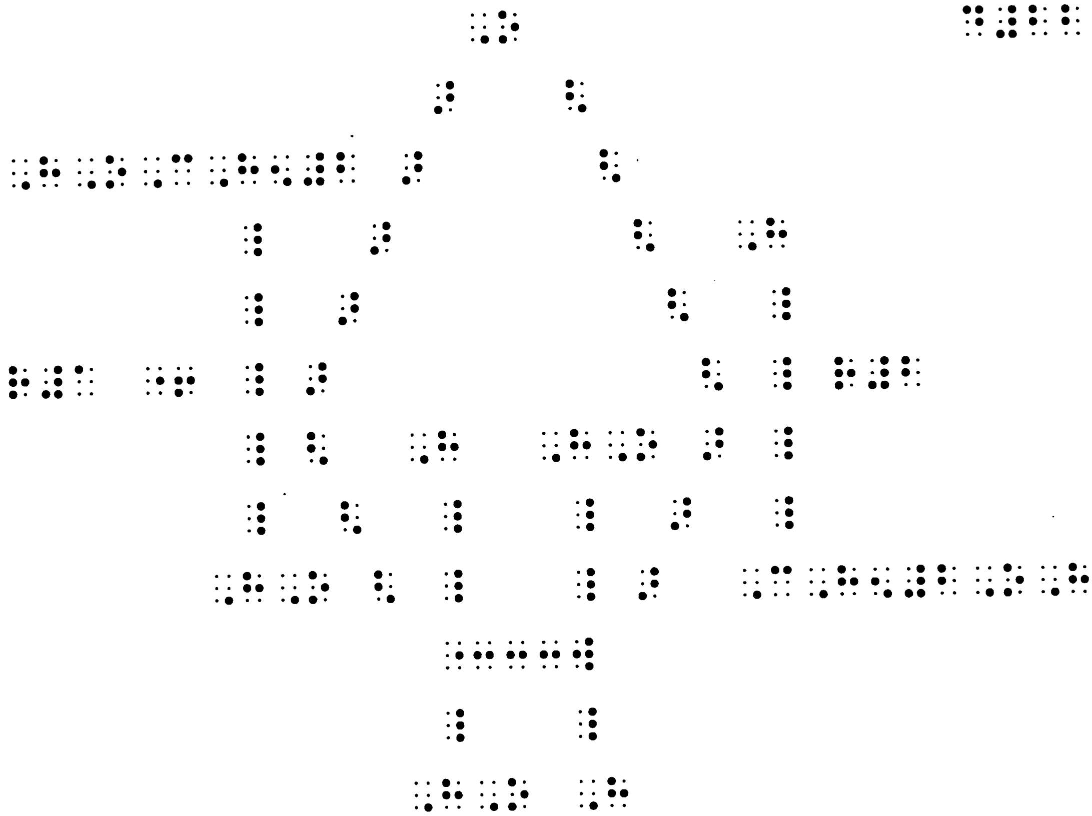
Note: UEBC does not yet provide for the bold line or for the wedge.

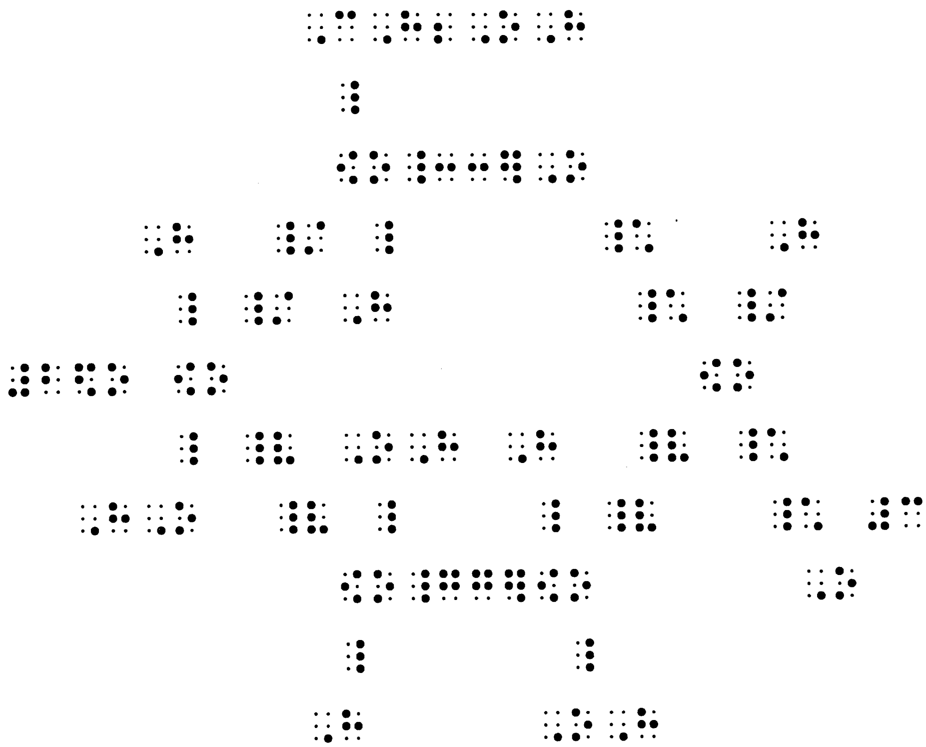


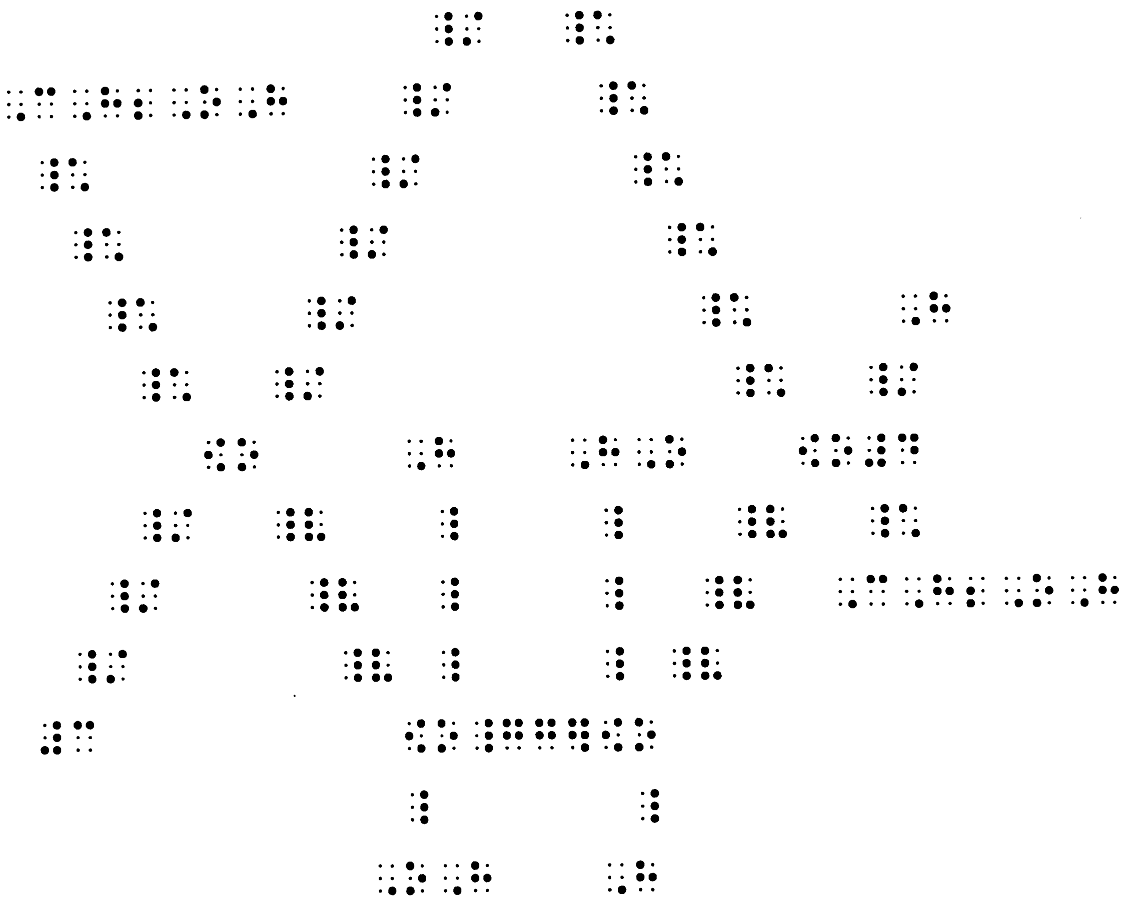


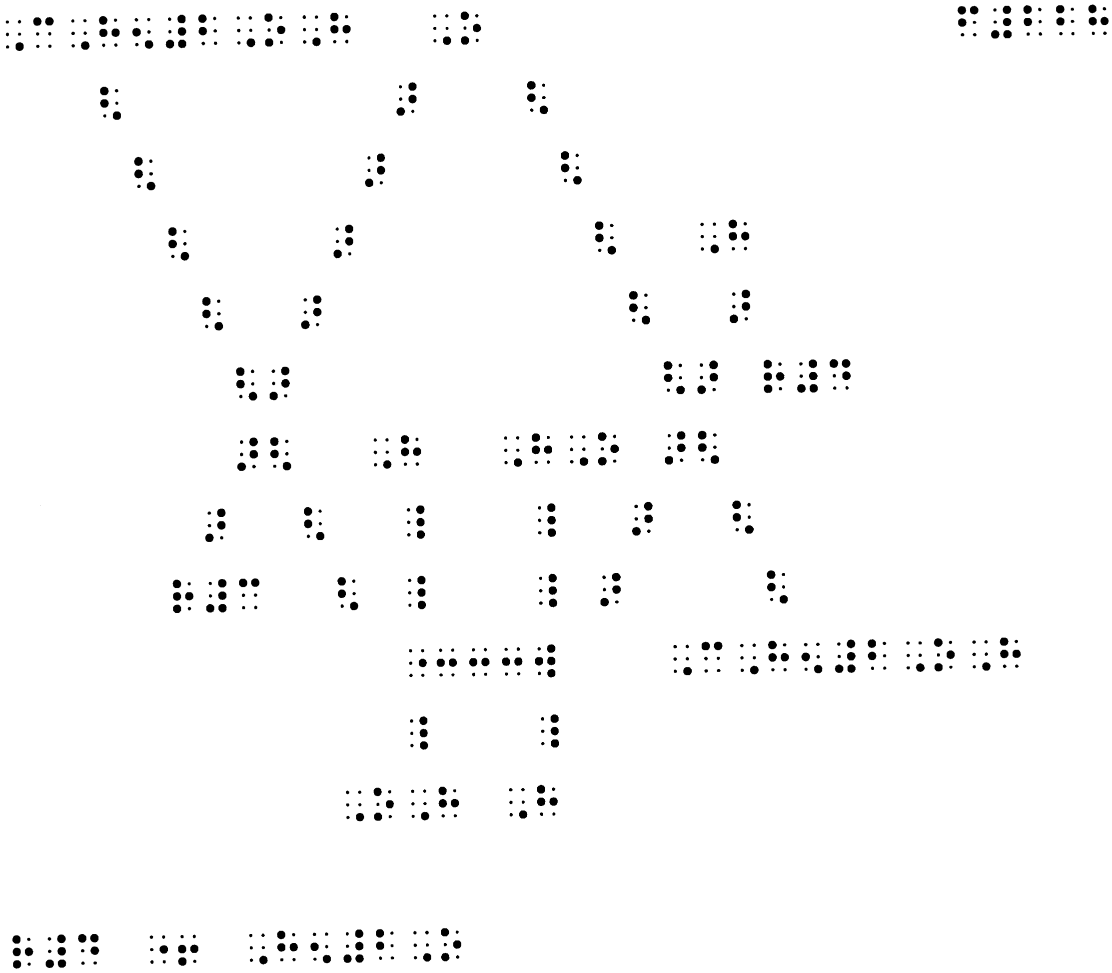


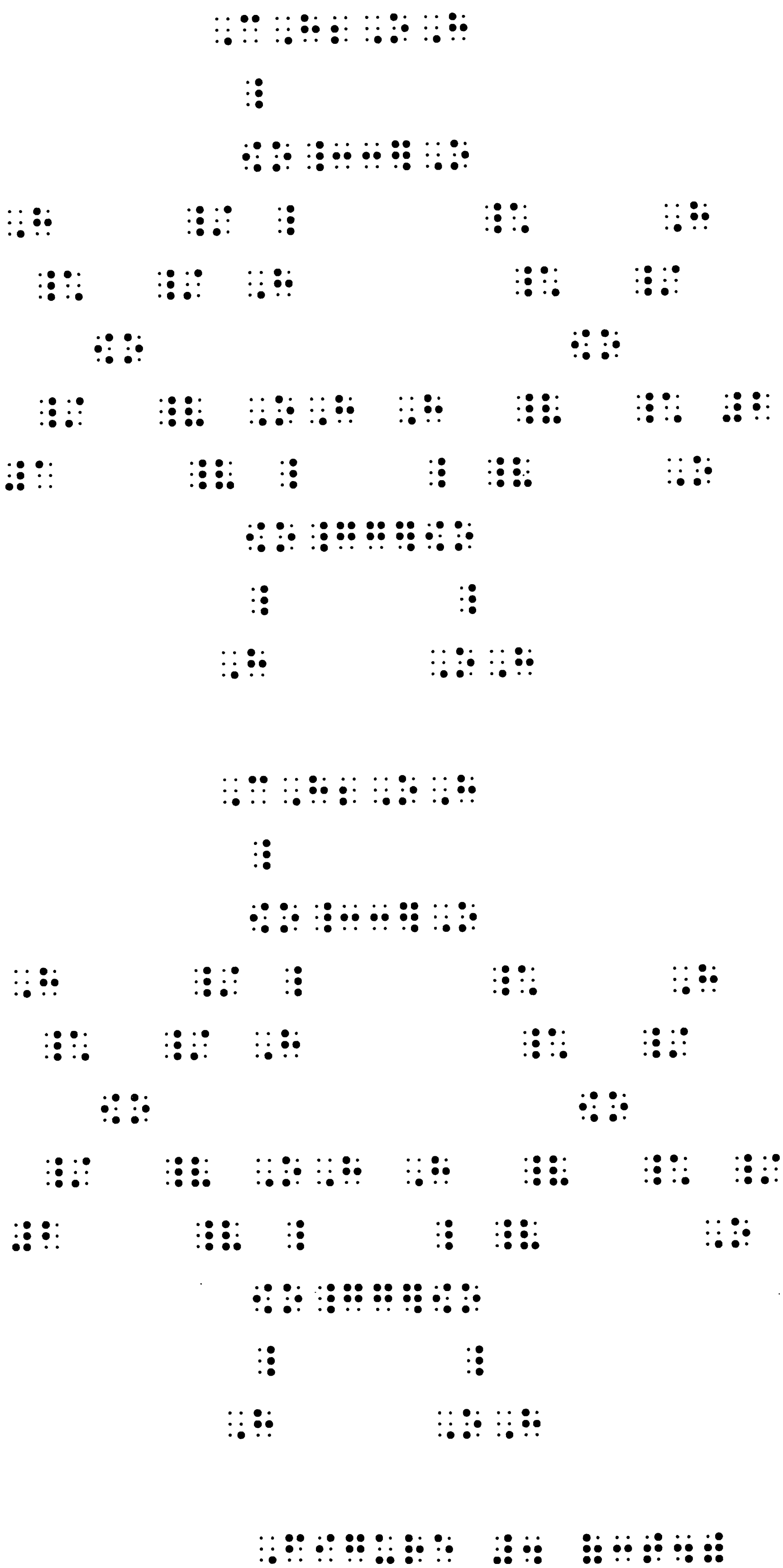


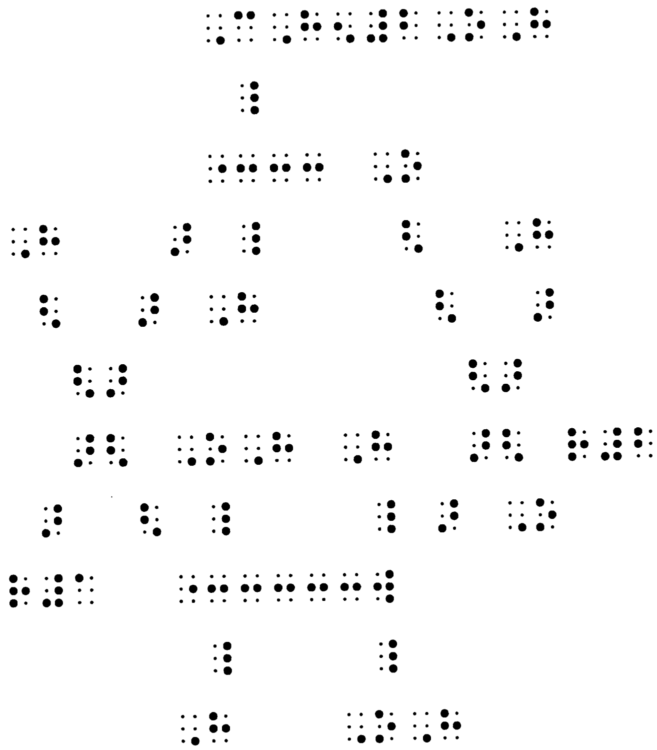


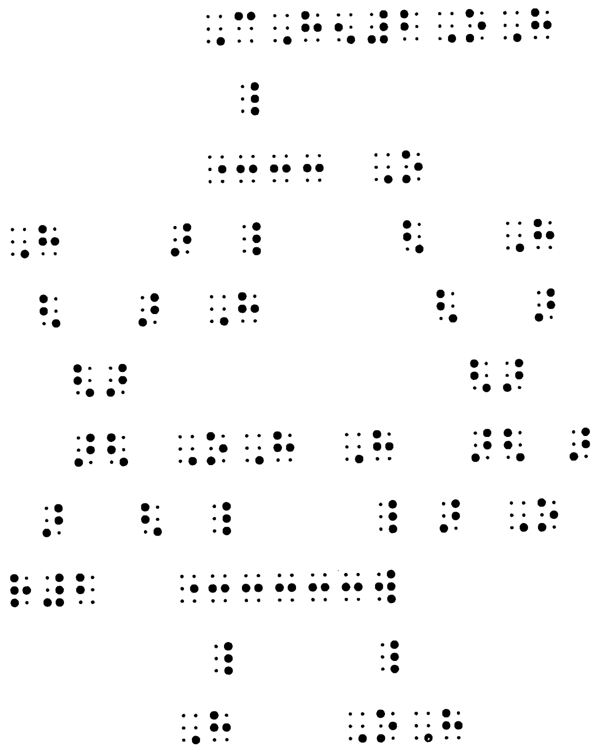


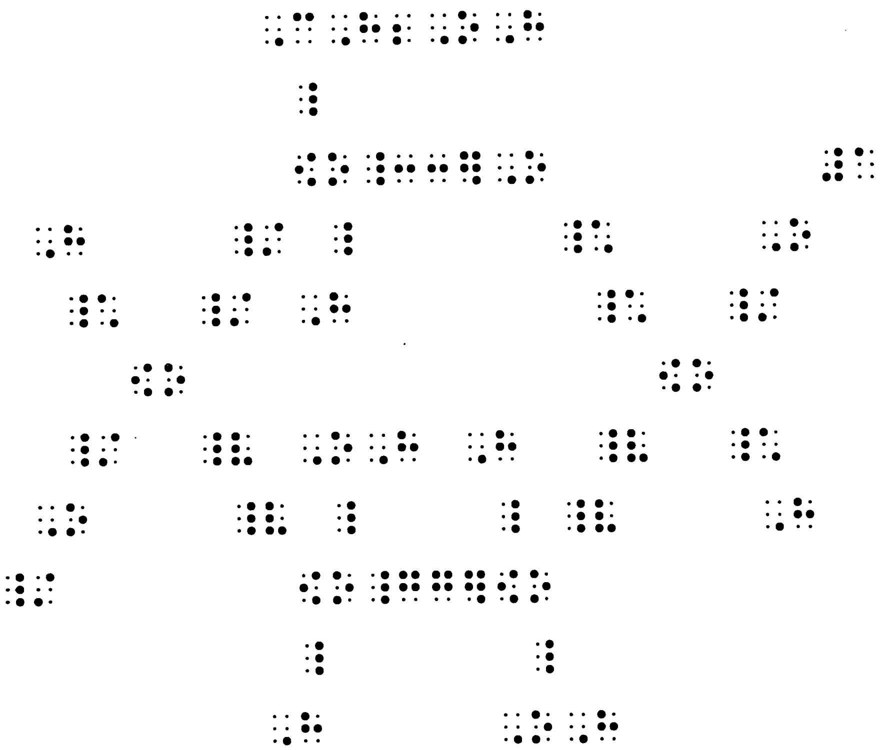


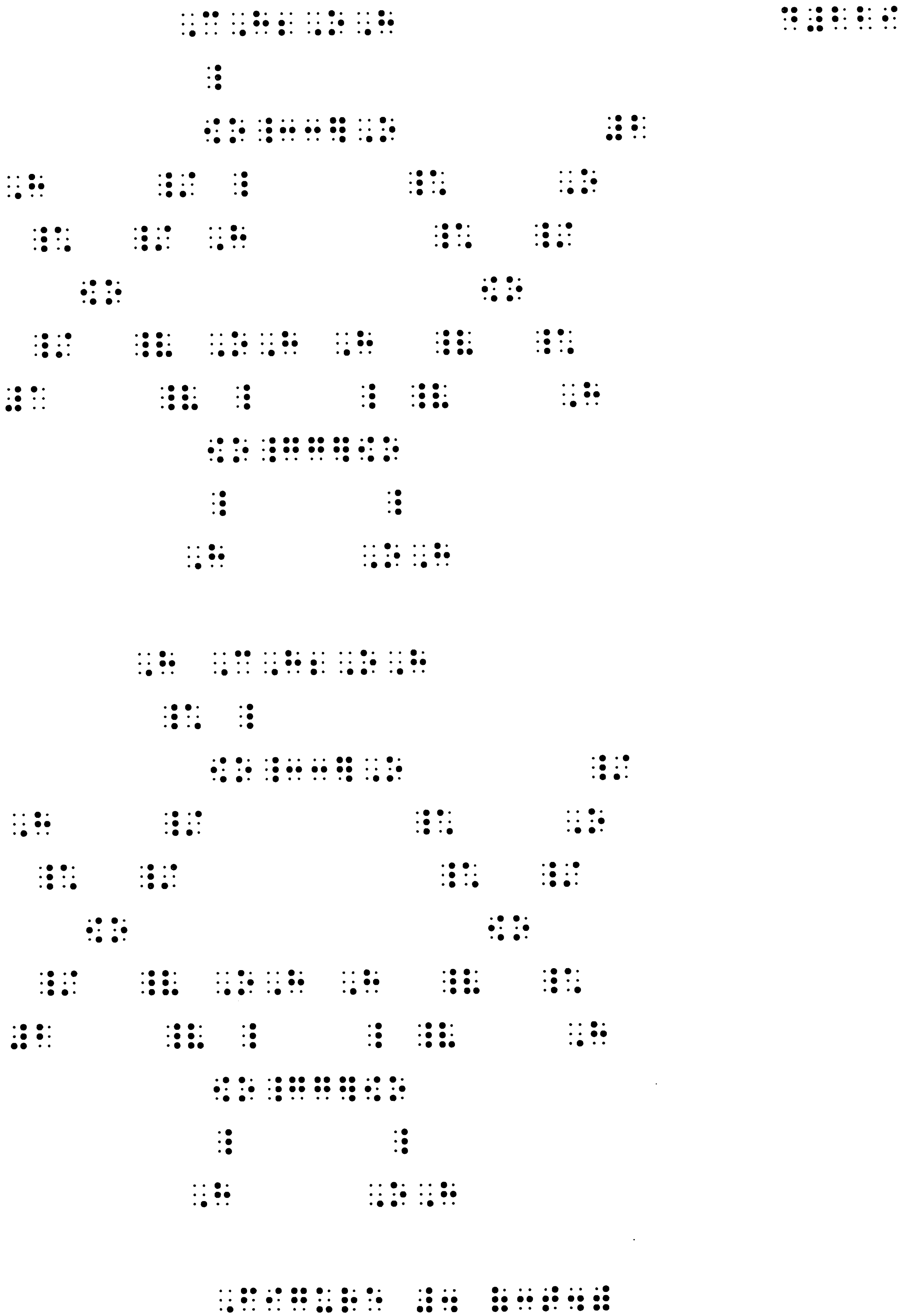


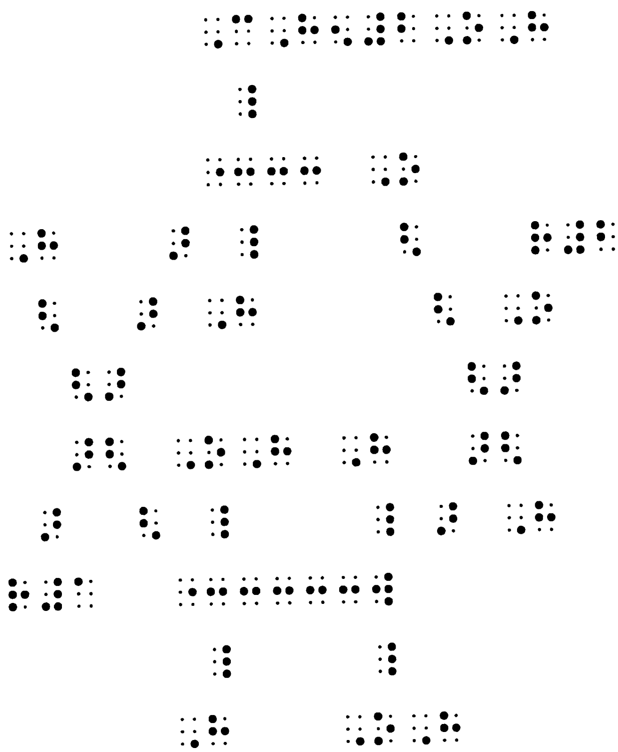


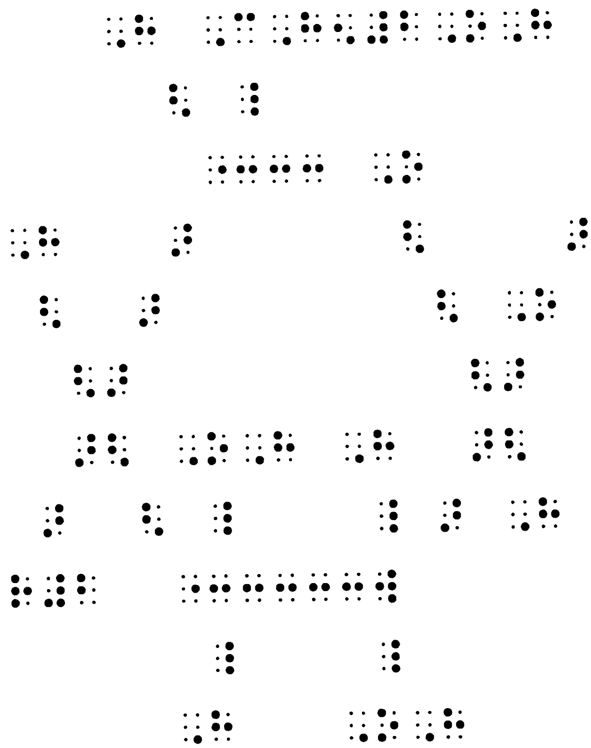


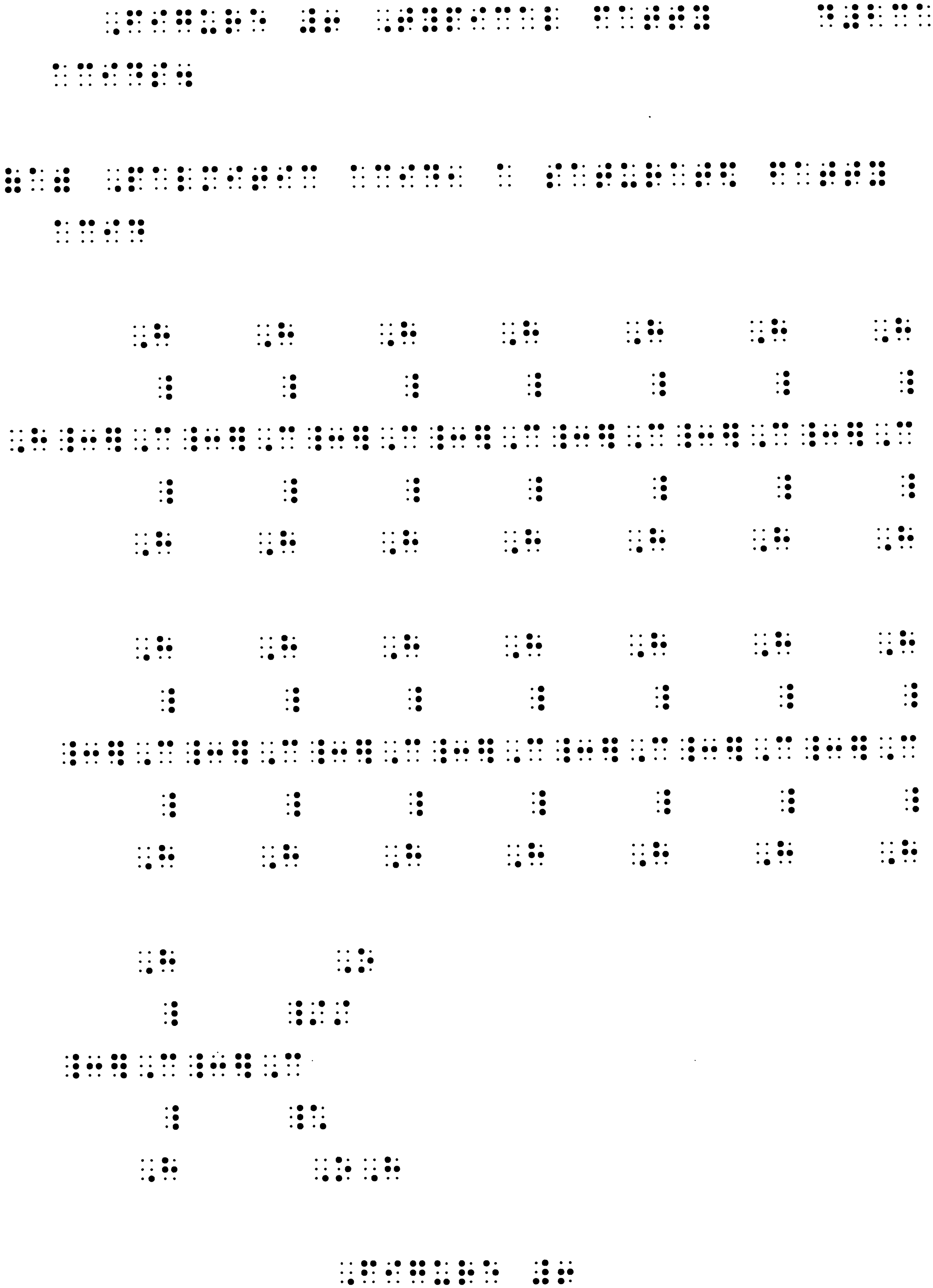












⠠⠋⠊⠛⠥⠗⠑ ⠼⠋ ⠠⠞⠽⠏⠊⠉⠁⠇ ⠋⠁⠞⠞⠽ ⠙⠼⠃⠉⠁
⠁⠉⠊⠙⠎⠲

⠐⠣⠁⠐⠜ ⠠⠏⠁⠇⠍⠊⠞⠊⠉ ⠁⠉⠊⠙⠂ ⠁ ⠎⠁⠞⠥⠗⠁⠞⠫ ⠋⠁⠞⠞⠽
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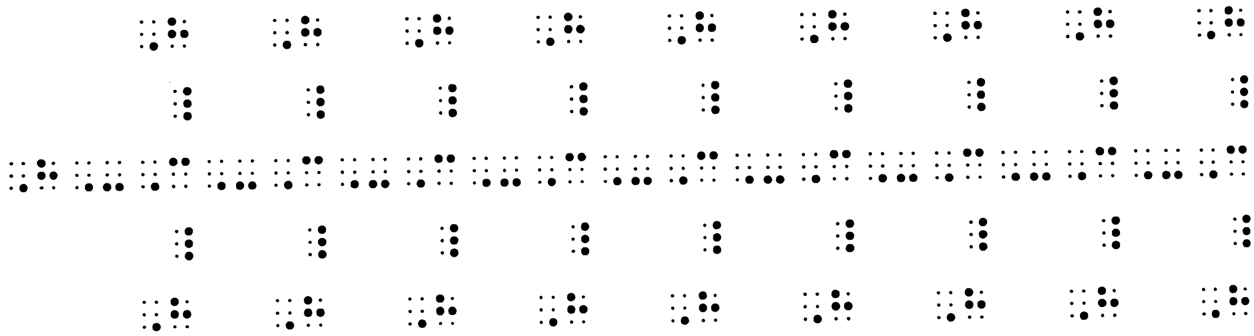

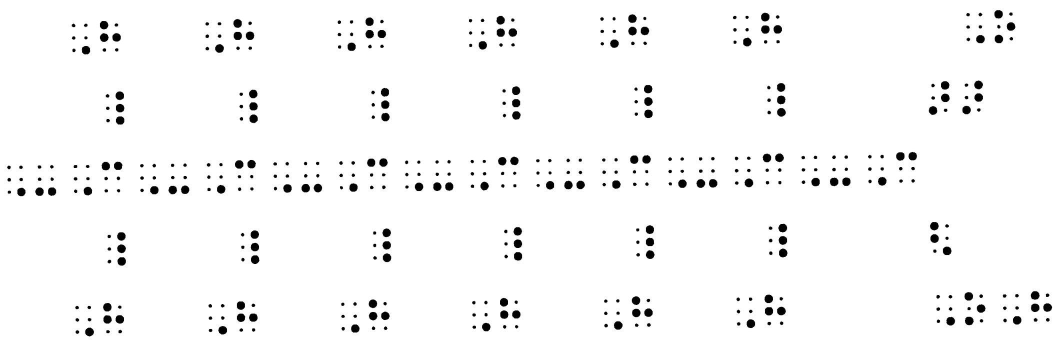

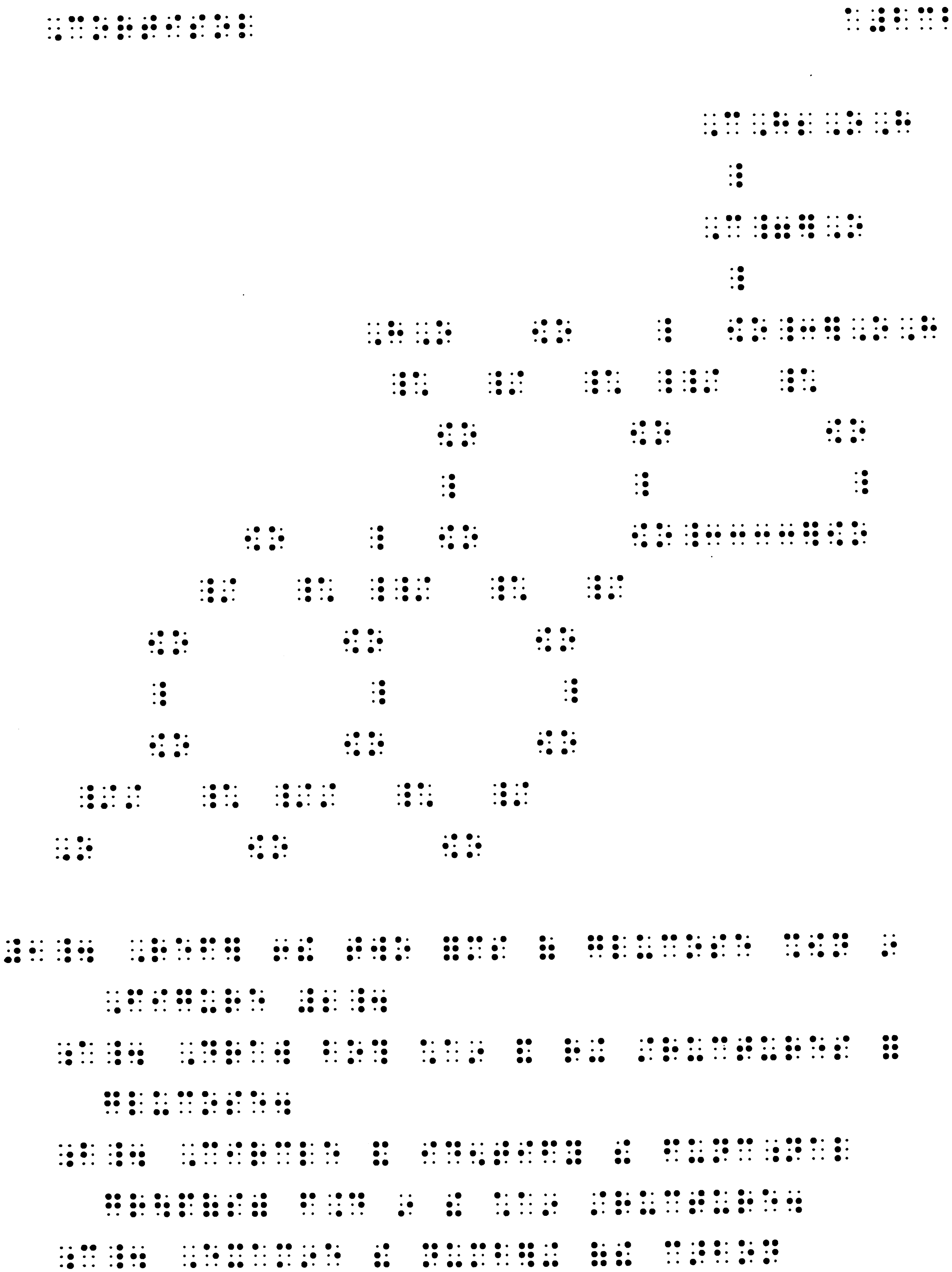

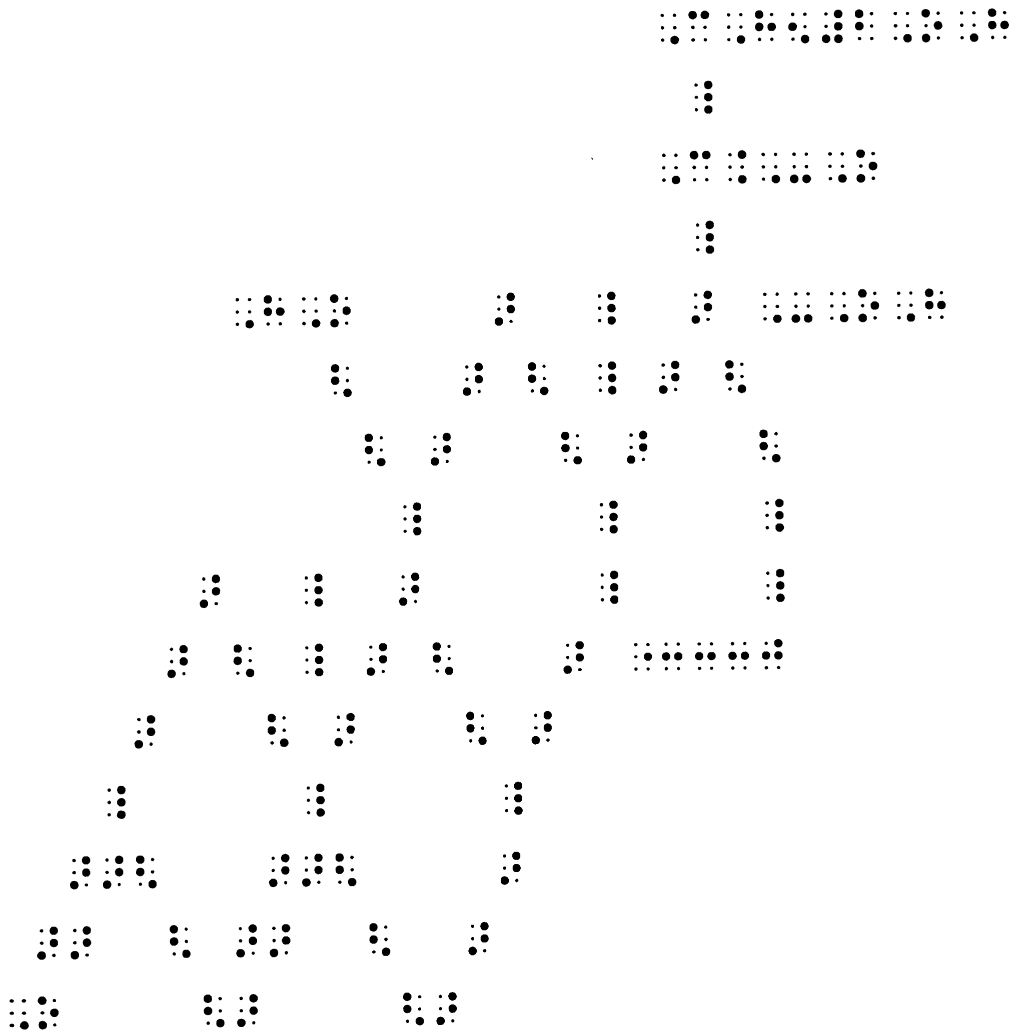

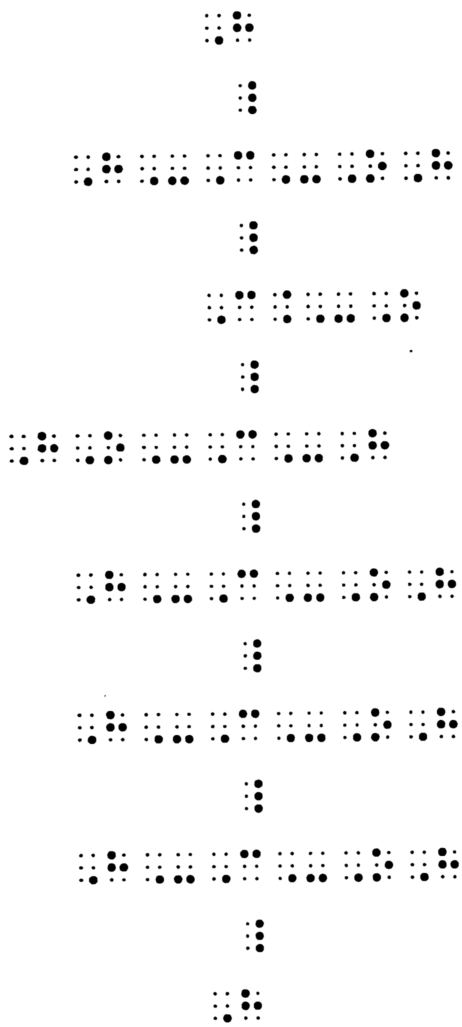

The term *polyunsaturated*, often used in food advertisements means that the food contains fats with two or more carbon-carbon double bonds in each fatty acid molecule. The term has become familiar because increasing evidence suggests that saturated fats may contribute to heart problems, while some unsaturated fats may not. Saturated fats are associated with formation of plaque (fat[illegible] fibrous matter), which can block arteries. The result is a condition known as "hardening of the arteries," or atherosclerosis, a particular threat to coronary (heart) arteries and arteries leading to the brain. If coronary arteries are blocked, a heart attack can result, damaging the heart muscle.

END OF CHEMISTRY SAMPLE IN UEBC