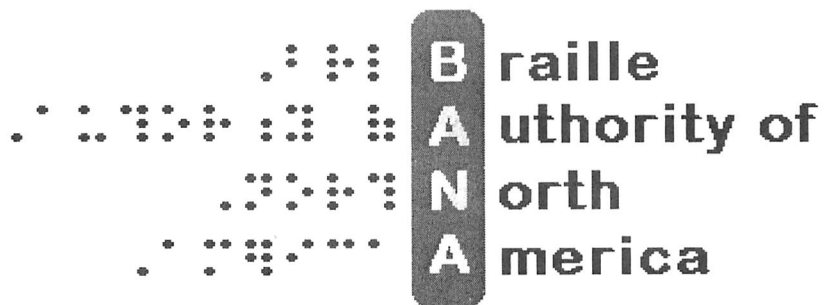


BANA
UEBC SAMPLER 2

Prepared under the auspices of the
Braille Authority of North America

June 2001



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Introduction

The Braille Authority of North America (BANA) is pleased to send you UEBC Sampler 2. It illustrates the draft braille code being developed by the Unified English Braille Code Research Project under the direction of the International Council on English Braille (ICEB). If you have not seen UEBC Sampler 1, please contact the appropriate person from the list of addresses at the end of this Introduction.

UEBC Sampler 2 includes examples of published technical materials. Four of them, each 10 consecutive print pages, are brailled in UEBC and in the BANA code in use in North America for that subject, e.g. Nemeth Code for the algebra and calculus, Computer Code for the computer notation, and Chemistry Code for the chemistry. There are also: some elementary arithmetic samples; a list of examples used in her teaching by Susan Osterhaus, a mathematics teacher at the Texas School for the Blind and Visually Impaired; and an example provided by Jane Corcoran, a California transcriber.

At the beginning of each sample there is a list of the new and changed symbols you will encounter in UEBC. Each symbol is preceded by the dot locator (dots 46, 123456).

To help those who may not be familiar with the three present BANA technical codes, a list of symbols required to read the materials accompanies each sample.

The samples are organized so that the UEBC version is presented on the right-hand page with the corresponding material in the appropriate BANA technical code presented on the left-hand facing page.

UEBC format has not been finalized. These examples follow the same format used by the present BANA codes with one

exception. UEBC follows the spacing of the original text. For instance, mathematical signs of operation are spaced or unspaced as they appear in print.

In the narrative portion of these examples you will notice that eight contractions found in English Braille American Edition (EBAE) are not used in UEBC. Six are brailled letter-for-letter: **ble, com, dd, ally, to, by**; and two are brailled with a contraction: **into** and **ation**. All of the other 181 contractions, wordsigns and shortforms are unchanged.

Each UEBC symbol is unambiguous—a print symbol is represented by the same braille symbol regardless of the subject. Because there are only 63 possible single cell dot combinations many symbols have to be made up of two or three cells. If the braille symbol uses the same dot configuration as a contraction it must be preceded by a grade one indicator to remove any ambiguity. This design feature will make it easier for a braille reader to work independently using computer translation and be assured of the accuracy of both the print and braille.

Nemeth Reading Notes

Nemeth numbers are brailled in the lower part of the cell. The number indicator is omitted when numbers immediately follow other symbols. Number indicators are also omitted when numbers are aligned one above the other, as in addition problems. A punctuation indicator is used where necessary to distinguish signs of punctuation from digits.

UEBC Reading Notes

UEBC uses grade one indicators to set off sections of text containing symbols that would otherwise be read as literary contractions. The number indicator also sets grade one mode for unspaced symbols or letters that follow a number.

Print Copies

To obtain a print copy of the examples in Sampler 2 or a print copy of Sampler 1 contact

The American Foundation for the Blind
National Literacy Center
Contact: Frances Mary D'Andrea
404-525-2303 or AFB's Information Center 800-232-5463
E-mail: literacy@afb.net

To obtain a braille copy of Sampler 1 contact

Kim Charlson, Braille Authority of North America,
617-972-7249 or e-mail charlsonk@perkins.pvt.k12.ma.us

For Canadians, print and braille copies of Sampler 1 may be obtained by contacting

Darleen Bogart
The Canadian National Institute for the Blind
416-480-7530 or 800-268-8818
E-mail: bogartd@lib.cnib.ca

Questionnaire

Thank you for reading UEBC Sampler 2. BANA wants your comments and has prepared a short list of questions as a guide (to get you started). You may respond in braille, print, on audio tape or by e-mail. Up-to-date information on UEBC is available by visiting the BANA web site at <http://www.brailleauthority.org>

Directions: Please complete the following questionnaire after reading through Sampler 2. Write your answers below, on a separate sheet of paper, or in an e-mail message. Send your completed survey to:

Warren Figueiredo
Louisiana Instructional Materials Center for the Blind
1230 Government Street
Baton Rouge, LA 70802
E-mail responses: UEBCinput@aol.com

Your input will be useful to the BANA Board in studying the UEBC. Thank you!

Questions about UEBC and Other Issues

1. Before you read Sampler 2, what were your feelings toward unifying the braille codes?
2. Having read through Sampler 2, how have your ideas/feelings changed toward a unified braille code?
3. What do you really like about the Unified English Braille Code (UEBC)?
4. What do you really dislike about the UEBC?
5. What issues do you feel the BANA Board should consider in making a decision on adoption of the UEBC?
6. Are your comments based on all six samples? Yes No
If not, please check those which are included. 1 2 3 4 5 6

Optional Background Information

7. What kind of materials do you typically read in braille?
Check all that apply.
 - a. magazines
 - b. fiction
 - c. nonfiction
 - d. textbooks
 - e. other (list):
8. What math or technical materials do you read? Check all that apply.
 - a. textbooks
 - b. technical materials for my job
 - c. computer braille
 - d. chemistry or other scientific notation
 - e. other (list):

9. What math code did you learn in school?
 - a. upper numbers (Taylor Code)
 - b. lower numbers (Nemeth)
 - c. upper numbers (BAUK—Braille Authority of the United Kingdom)
 - d. no math code used

10. When did you learn braille?
 - a. before age 18
 - b. 18-55 years of age
 - c. 56 and above

Optional Personal Information

Name:

Age:

Gender:

Occupation:

Preferred Literacy Medium:

E-mail address:

Address:

Telephone:

Sample 1. Spatially Arranged Mathematics

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).



1a. Arithmetic Problems

Add.

$$\begin{array}{r} 28. \quad 8 \\ + 5 \\ \hline \end{array} \quad \begin{array}{r} 29. \quad 5 \\ + 8 \\ \hline \end{array} \quad \begin{array}{r} 30. \quad 2 \\ + 7 \\ \hline \end{array} \quad \begin{array}{r} 31. \quad 7 \\ + 2 \\ \hline \end{array} \quad \begin{array}{r} 32. \quad 3 \\ + 0 \\ \hline \end{array} \quad \begin{array}{r} 33. \quad 8 \\ + 0 \\ \hline \end{array}$$

$$34. (2 + 6) + 5 \quad 35. 2 + (6 + 5) \quad 36. (7 + 3) + 0 \quad 37. 4 + 5 + 8$$

Find the sums and differences. Use a calculator if necessary.

$$\begin{array}{r} 1. \quad 2,964 \\ + 5,682 \\ \hline \end{array} \quad \begin{array}{r} 2. \quad 6,587 \\ + 2,744 \\ \hline \end{array} \quad \begin{array}{r} 3. \quad 4,532 \\ + 1,607 \\ \hline \end{array} \quad \begin{array}{r} 4. \quad 4,430 \\ - \quad 726 \\ \hline \end{array} \quad \begin{array}{r} 5. \quad 6,429 \\ - 5,161 \\ \hline \end{array} \quad \begin{array}{r} 6. \quad 7,000 \\ - 2,674 \\ \hline \end{array}$$







Find the products.

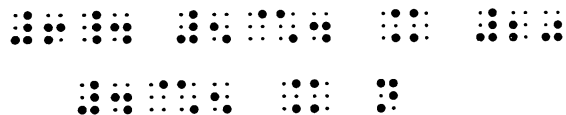
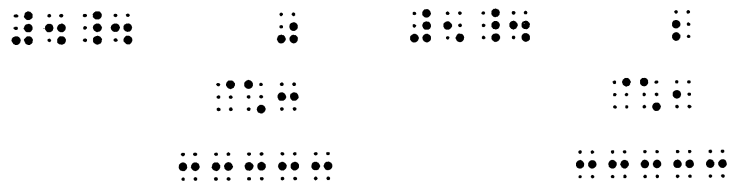
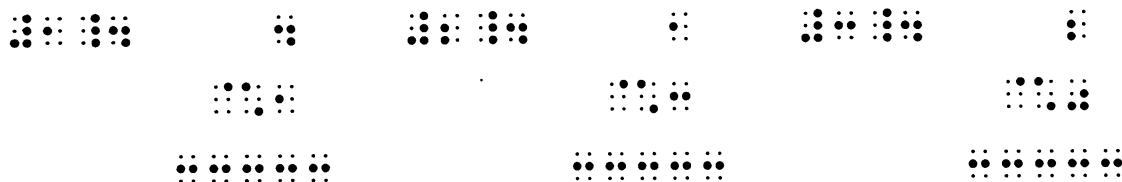
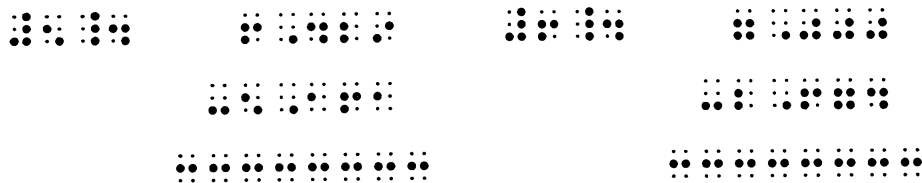
$$\begin{array}{r} 1. \quad 4 \\ \times 1 \\ \hline \end{array} \quad \begin{array}{r} 2. \quad 1 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 3. \quad 2 \\ \times 0 \\ \hline \end{array} \quad \begin{array}{r} 4. \quad 0 \\ \times 3 \\ \hline \end{array} \quad \begin{array}{r} 5. \quad 2 \\ \times 1 \\ \hline \end{array} \quad \begin{array}{l} 6. \quad 5 \times 4 = 20 \\ \quad 4 \times 5 = n \end{array}$$

Arithmetic Problems in Nemeth Code

Nemeth Symbols	
⠄	punctuation indicator
⠐	plus sign
⠐	minus sign
⠈⠐	times sign (cross)
⠈⠐	equal sign
⠐	opening parenthesis
⠐	closing parenthesis
⠂	mathematical comma

Arithmetic Problems in UEBC

UEBC Symbols	
	plus sign
	minus sign
	times sign (cross)
	equal sign
	opening parenthesis
	closing parenthesis



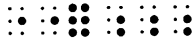







1b. System of Equations

Equating coefficients leads to the system

$$\begin{array}{rcl} c_1 + 2c_2 + c_3 & = & 0 \\ -2c_1 + c_2 + 8c_3 & = & 0 \\ 3c_1 + 8c_2 + 7c_3 & = & 0 \end{array}$$





System of Equations in UEBC

UEBC Symbols	
	begin grade one passage
	end grade one passage
	subscript next item
	plus sign
	minus sign
	equal sign







1c. Matrix Multiplication

For each fixed x in $[a, b]$, the matrix equation

$$\begin{pmatrix} f_1(x) & f_2(x) & \hat{e} & f_n(x) \\ f_1'(x) & f_2'(x) & \hat{e} & f_n'(x) \\ \ddot{E} & & & \\ f_1^{(n-1)}(x) & f_2^{(n-1)}(x) & \hat{e} & f_n^{(n-1)}(x) \end{pmatrix} \begin{pmatrix} \alpha_1 \\ \alpha_2 \\ \ddot{E} \\ \alpha_n \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ \ddot{E} \\ 0 \end{pmatrix} \quad (11)$$

Matrix Multiplication in Nemeth Code

Nemeth Symbols	
⠠	opening parenthesis
⠨	closing parenthesis
⠠⠠	multiline opening parenthesis
⠠⠨	multiline closing parenthesis
⠠⠠	opening bracket
⠠⠨	closing bracket
⠠	mathematical comma
⠠	subscript indicator
⠠	superscript indicator
⠠	return to baseline after subscript or superscript
⠠	prime sign
⠠	minus sign
⠠⠠	equal sign
⠠⠠	Greek alpha

Matrix Multiplication in UEBC

UEBC Symbols	
⠠⠠⠠⠠⠠⠠⠠⠠	begin grade one passage
⠠⠠⠠⠠	end grade one passage
⠠⠠	opening parenthesis
⠠⠠	closing parenthesis
⠠⠠	opening bracket
⠠⠠	closing bracket
⠠⠠⠠	multiline opening parenthesis
⠠⠠⠠	multiline closing parenthesis
⠠	subscript next item
⠠	superscript next item
⠠⠠⠠	ellipsis
⠠⠠⠠⠠⠠⠠	vertical ellipsis
⠠	prime sign
⠠	begin compound item
⠠	end compound item
⠠⠠	minus sign
⠠⠠	equal sign
⠠⠠	Greek alpha



1d. Long Division

4. Bring down the next digit in the dividend. Repeat these steps until there are no digits left to bring down.

$$\begin{array}{r} 620 \text{ R}19 \\ 32 \overline{)19,859} \\ \underline{-192} \\ 65 \\ \underline{-64} \\ 19 \\ \underline{-0} \\ 19 \end{array}$$

Remember to
check the answer.

Long Division in Nemeth Code

Nemeth Symbols

⋮	punctuation indicator
⋮	baseline indicator
⋮	curved division sign
⋮	mathematical comma
⋮	minus sign

Long Division in UEBC

UEBC Symbols



curved division sign



minus sign



numeric space



1e. Long Multiplication

2. Multiply by the tens digit.

$$\begin{array}{r} 1 \\ 643 \\ \times 35 \\ \hline 3215 \\ 19290 \leftarrow 30 \times 643 \\ \uparrow \end{array}$$

Remember to regroup.

You do not have to write the zero. Start the answer in the tens place.






1f. Cancelled Digits

Subtract: $16 - 3.98$

$$\begin{array}{r} 5\overset{9}{\cancel{10}} \\ 1\cancel{6}.\cancel{0}\cancel{0} \\ - 3.98 \\ \hline 12.02 \end{array}$$

Cancelled Digits in UEBC

UEBC Symbols	
	minus sign
	line through previous item
	decimal point












 








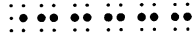








1g. Cancelled Fractions

$$\frac{3}{8} \times 1\frac{5}{9} = \frac{\cancel{3}^1}{\cancel{8}_4} \times \frac{\cancel{14}^7}{\cancel{9}_3}$$
$$= \frac{7}{12}$$

Cancelled Fractions in UEBC

UEBC Symbols	
	times sign (cross)
	equal sign
	line through previous item

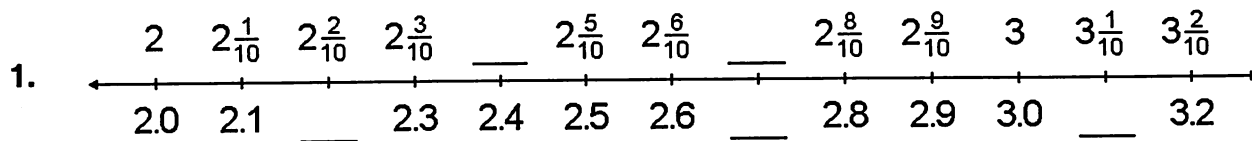
  



1h. Number Line

Use fractions, mixed numbers, and decimals to name points on the number line. Write each missing number.



Number Line in Nemeth Code

Nemeth Symbols	
⠄	punctuation indicator
⠠	left arrowhead
⠠⠠⠠	horizontal shaft of number line
⠨	vertical mark
⠡	right arrowhead
⠠⠠⠠	opening mixed fraction indicator
⠠⠠⠠	closing mixed fraction indicator
⠠	horizontal fraction line
⠨	missing number (blank underline)

Number Line in UEBC

UEBC Symbols	
⠠	grade one symbol
⠠⠠	left arrowhead
⠠⠠⠠	horizontal shaft of number line
⠠	vertical mark
⠠	(at end of line) continuation indicator
⠠⠠	right arrowhead
⠠⠠	dash (used for blank underline)
⠠	decimal point



1i. Addition Puzzle

Each letter in these problems represents a different digit.

1. What is the value of C?
2. What is the value of D?

$$\begin{array}{r} 8789 \\ 3BA7 \\ 482A \\ +7AB5 \\ \hline 2C287 \end{array}$$

$$\begin{array}{r} DEFF \\ -E2F6 \\ \hline 1997 \end{array}$$



Sample 2. Algebra

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).

3-4 SPECIAL PRODUCTS

Use synthetic division to find the quotient and remainder in the following divisions and express the answer in the form of Eq. (3-9).

9. $x^3 - 2x^2 + 3x - 4$ by $x - 3$

10. $2x^3 + x^2 - x + 4$ by $x + 1$

11. $x^4 - 5x^3 + x^2 - 6$ by $x - 1$

12. $x^3 + 3x^2 - 2x - 5$ by $x + 2$

13. Find the value of $x^3 - 2x^2 + 3x - 4$ at $x = 3$.

14. Find the value of $2x^3 + x^2 - x + 4$ at $x = -1$.

15. Find the value of $x^4 - 5x^3 + x^2 - 6$ at $x = 1$.

16. Find the value of $x^3 + 3x^2 - 2x - 5$ at $x = -2$.

17. Compare the values obtained in Problems 13 through 16 with the remainders in Problems 9 through 12.

18. a) Evaluate $ax^2 + bx + c$ at $x = p$.

b) Divide $ax^2 + bx + c$ by $x - p$.

c) Compare the remainder found in (b) with the value obtained in (a).

d) Would you expect the same type of result for any degree polynomial?

► **3-4 SPECIAL PRODUCTS**

There are certain special products which occur so frequently in algebra that they have been classified. These are given below. *The letters in the formulas may stand for any algebraic expression.* Each is a direct result of the axioms in Chapter 2. The reader should not only verify each by actually carrying out the steps and giving the reasons, but also memorize them, so that he can recognize both the product from the factors and the factors from the product.

$$a(x + y) \equiv ax + ay. \quad (3-11)$$

$$(x + y)(x - y) \equiv x^2 - y^2. \quad (3-12)$$

$$(x \pm y)^2 \equiv x^2 \pm 2xy + y^2. \quad (3-13)^*$$

$$(x + a)(x + b) \equiv x^2 + (a + b)x + ab. \quad (3-14)$$

$$(ax + b)(cx + d) \equiv acx^2 + (ad + bc)x + bd. \quad (3-15)$$

$$(x \pm y)^3 \equiv x^3 \pm 3x^2y + 3xy^2 \pm y^3. \quad (3-16)$$

$$(x \pm y)(x^2 \mp xy + y^2) \equiv x^3 \pm y^3. \quad (3-17)$$

The reader should determine which of the above formulas is used in the following illustrations.

* The sign \pm is read "plus or minus." If the upper (lower) sign is used in the left member, it is also used in the right, so that $(x \pm y)^2 \equiv x^2 \pm 2xy + y^2$ means $(x + y)^2 \equiv x^2 + 2xy + y^2$ and $(x - y)^2 \equiv x^2 - 2xy + y^2$.

EXTENSIONS OF THE LOGIC OF ALGEBRA 3-4

$$\begin{aligned}\text{Illustration 1. } (2x^2 - 3y)(2x^2 + 3y) &\equiv (2x^2)^2 - (3y)^2 \\ &\equiv 4x^4 - 9y^2.\end{aligned}$$

$$\begin{aligned}\text{Illustration 2. } (x + 2)(x + 5) &\equiv x^2 + (2 + 5)x + 10 \\ &\equiv x^2 + 7x + 10.\end{aligned}$$

$$\begin{aligned}\text{Illustration 3. } (3x + 4y)(2x - 3y) &\equiv 6x^2 + (-9 + 8)xy - 12y^2 \\ &\equiv 6x^2 - xy - 12y^2.\end{aligned}$$

Illustration 4

$$\begin{aligned}(x + y - 1)^3 &\equiv [(x + y) - 1]^3 \\ &\equiv (x + y)^3 - 3(x + y)^2 + 3(x + y) - 1 \\ &\equiv x^3 + 3x^2y + 3xy^2 + y^3 - 3x^2 - 6xy - 3y^2 + 3x + 3y - 1.\end{aligned}$$

Here $(x + y)$ is considered first as one term.

Illustration 5

$$\begin{aligned}(3x + 2y)(9x^2 - 6xy + 4y^2) &\equiv (3x + 2y)[(3x)^2 - (3x)(2y) + (2y)^2] \\ &\equiv (3x)^3 + (2y)^3 \\ &\equiv 27x^3 + 8y^3.\end{aligned}$$

PROBLEMS

Find the following products.

1. $2a(3x - 4y)$
2. $-3x(2x + 7y)$
3. $-7xy(3x^2 + 4y)$
4. $4x^2yz(z^2 + xy + yz)$
5. $(2x - 3y)(2x + 3y)$
6. $(7x + 5y^2)(7x - 5y^2)$
7. $(x + 2y)(x - 2y)(x^2 + 4y^2)$
8. $(x - 3)^2$
9. $(2x + 7y)^2$
10. $(3x^2y - 5z^2)^2$
11. $(x - 2)(x - 5)$
12. $(2x + 3)(x - 5)$
13. $(xy^2 - z^2w)^2$
14. $(\frac{1}{2}x + \frac{3}{8}y)^2$
15. $(4x - 3y)(7x + 3y)$
16. $[(x + 1) - z][(x + 1) + z]$
17. $(2x + 3y + 3)(2x + 3y - 3)$
18. $(2x + 3y + 4z)^2$
19. $(x - 2y - z)^2$
20. $(2a + b)^3$
21. $(x + 2)(x^2 - 2x + 4)$
22. $(x - 3)(x^2 + 3x + 9)$
23. $(x + 3y + 2z - 4w)(x + 3y - 2z + 4w)$
24. $(4x - 2y - 3z + 3w)(4x + 2y + 3z + 3w)$
25. $(a - b + c - d)^2$
26. $(2a + 3b - c - 4d)^2$
27. $[2(x + 2y) - 3][2(x + 2y) + 4]$
28. $[2(x - 3y) + 5][3(x - 3y) - 2]$
29. $(2x + 3y)^3$
30. $(5x - 3y)^3$

3-5 FACTORS AND FACTORING

The process of factoring an algebraic expression is similar to that of finding the factors of a composite number. Recall the discussion of prime and composite integers in Article 1-4. This process, which is usually restricted at this elementary stage to factoring polynomials with rational coefficients and to factors completely free from irrational numbers, is frequently performed by reversing the processes considered in Article 3-4. Such a factorization is considered complete when each algebraic factor is a *prime factor*; that is, an algebraic expression that cannot be factored without violating the above restrictions.

The more common types of factoring are illustrated below. Note the importance and application of the distributive axioms in this discussion.

Example 1. Factor $2ax^2 - 4ay^2 + 8a^2x$.

Solution. The polynomial in this problem has $2a$ as a common factor.

$$2ax^2 - 4ay^2 + 8a^2x \equiv 2a(x^2 - 2y^2 + 4ax).$$

Example 2. Factor $x(a + 2b) - 3y(a + 2b)$.

Solution. Each of the two expressions has the common term $(a + 2b)$. Therefore,

$$x(a + 2b) - 3y(a + 2b) \equiv (x - 3y)(a + 2b).$$

Example 3. Factor $(4x^2/y^2) - (9a - b)^2$.

Solution. This expression is the difference between two perfect squares.

$$\begin{aligned} \frac{4x^2}{y^2} - (9a - b)^2 &\equiv \left(\frac{2x}{y}\right)^2 - (9a - b)^2 \\ &\equiv \left[\frac{2x}{y} + (9a - b)\right] \left[\frac{2x}{y} - (9a - b)\right] \\ &\equiv \left(\frac{2x}{y} + 9a - b\right) \left(\frac{2x}{y} - 9a + b\right). \end{aligned}$$

Example 4. Factor $9x^2 - 30xy + 25y^2$.

Solution. This algebraic expression is a perfect square.

$$9x^2 - 30xy + 25y^2 \equiv (3x - 5y)^2.$$

Example 5. Factor $27x^3 + (8/y^3)$.

Solution. The algebraic expression is the sum of two cubes. Accordingly,

$$27x^3 + \frac{8}{y^3} \equiv \left(3x + \frac{2}{y}\right) \left(9x^2 - \frac{6x}{y} + \frac{4}{y^2}\right).$$

EXTENSIONS OF THE LOGIC OF ALGEBRA 3-5

Example 6. Factor $12x^2 + 7xy - 10y^2$.

Solution. This trinomial in the form of Eq. (3-15) is factored by trial and error. The result will be in the form $(ax + by)(cx + dy)$, where $ac = 12$, $bd = -10$, and $ad + bc = 7$. Here a and c are both plus, and b and d are different in sign. The correct combination, we find, is $12x^2 + 7xy - 10y^2 \equiv (4x + 5y)(3x - 2y)$.

Example 7. Factor $6x^4 + 7x^2y^2 - 3y^4$.

Solution. This is the same type as Example 6.

$$6x^4 + 7x^2y^2 - 3y^4 \equiv (3x^2 - y^2)(2x^2 + 3y^2).$$

Although the first factor on the right is the difference of two squares, it cannot be factored further, for such factorization would introduce irrational quantities.

PROBLEMS

Factor the following completely.

1. $4x - 20$
2. $10x + 15yz$
3. $3y^2 - 9y$
4. $4x^3y^2 + 6x^2y^3$
5. $xy^2z^3 - 3x^2yz^2 + 5xy^3z^2$
6. $a^2b^3c^4 - a^3b^4c^5 + 2a^2b^4c^4$
7. $3y(2x + 5) - 4x(2x + 5)$
8. $3y(4 - y) + 6x^2(4 - y)$
9. $2z^2(x + 3y) - 6xz(x + 3y)$
10. $3x(3 - 2y) - 2xy(3 - 2y)$
11. $9 - a^2$
12. $16x^2 - 9y^2$
13. $225a^8 - 64b^2$
14. $(c^6/d^8) - 121$
15. $x^3y^4 - 25xd^6$
16. $0.01x^4 - 196y^8$
17. $(x + 2y)^2 - z^2$
18. $(3x - 2y)^2 - 25z^2$
19. $(a + b)^2 - (c + d)^2$
20. $9(2x - y)^2 - 4(2a + b)^2$
21. $81(4x - 3y)^2 - 25(3z + w)^2$
22. $x^2 + 6x + 9 - (y^2 + 4y + 4)$
23. $x^2 - 8x + 16$
24. $4a^2 - 12ab + 9b^2$
25. $66xy + 9x^2y^2 + 121$
26. $2x^3 - 28x^2 + 98x$
27. $5z^2 - 30wz + 45w^2$
28. $x^{2n} + 2x^ny^n + y^{2n}$
29. $(3 - x)^2 + 8(3 - x) + 16$
30. $25 - 30(2x - 3y) + 9(2x - 3y)^2$
31. $a^3 - 8$
32. $1 + (8/x^9)$
33. $8x^{6n} + 27y^{3m}$
34. $x^3 - (y^3/64)$
35. $27(x - y)^3 - 8(x + y)^3$
36. $5(a - 2b)^3 - 625(a - 2b)^3$
37. $x^2 - 7x + 12$
38. $y^2 - 2y - 8$
39. $a^2b^2 - ab - 20$
40. $2x^2 + 8x + 6$
41. $35x^2 - 24x + 4$
42. $3y^2 - y - 10$
43. $6a^2 + 7a - 20$
44. $2x^2 - 23xy - 39y^2$

3-5 FACTORS AND FACTORING

45. $(x + y)^2 - 7(x + y) + 10$ 46. $(y + z)^2 + (y + z) - 42$
 47. $2(2x + y)^2 - (2x + y) - 10$
 48. $6(x + y)^2 + 5(x + y)(y + z) - 6(y + z)^2$
 49. $12(a + b)^2 - 14(a + b)(c + d) - 10(c + d)^2$
 50. $4(x - 2)^2 + 5(x - 2)(y + 4) - 21(y + 4)^2$

There are many algebraic expressions which, by proper grouping, can be put into one of the forms in the previous examples and then factored.

Example 8. Factor $ax - ay - bx + by$.

Solution. If, by the associative axiom, we group the first two terms together, and the last two together, and (use the distributive axiom) factor out the common term, we transform the expression into the form of Example 2.

$$\begin{aligned} ax - ay - bx + by &\equiv a(x - y) - b(x - y) \\ &\equiv (x - y)(a - b). \end{aligned}$$

Example 9. Factor $4x^3 - 12x^2 - x + 3$.

Solution. Again we group the first two terms and the last two terms.

$$\begin{aligned} 4x^3 - 12x^2 - x + 3 &\equiv 4x^2(x - 3) - (x - 3) \\ &\equiv (x - 3)(4x^2 - 1) \\ &\equiv (x - 3)(2x + 1)(2x - 1). \end{aligned}$$

In both these examples we could have grouped the first and third, and the second and fourth terms, and obtained the same result.

Example 10. Factor $4x^2 - 12xy + 9y^2 + 4x - 6y - 3$.

Solution. If we group the first three terms, the solution becomes clear.

$$\begin{aligned} 4x^2 - 12xy + 9y^2 + 4x - 6y - 3 &\equiv (2x - 3y)^2 + 2(2x - 3y) - 3 \\ &\equiv [(2x - 3y) + 3][(2x - 3y) - 1] \\ &\equiv (2x - 3y + 3)(2x - 3y - 1). \end{aligned}$$

Example 11. Factor $x^4 + 2x^2y^2 + 9y^4$.

Solution. If the coefficient of the second term were 6, the expression would be a perfect square. Therefore, if we add (and subtract) $4x^2y^2$, our solution becomes evident.

$$\begin{aligned} x^4 + 2x^2y^2 + 9y^4 &\equiv x^4 + 6x^2y^2 + 9y^4 - 4x^2y^2 \\ &\equiv (x^2 + 3y^2)^2 - (2xy)^2 \\ &\equiv (x^2 + 3y^2 + 2xy)(x^2 + 3y^2 - 2xy). \end{aligned}$$

EXTENSIONS OF THE LOGIC OF ALGEBRA 3-6

PROBLEMS

Factor the following expressions.

1. $ax - ay - by + bx$
2. $ax - 2ay - 6by + 3bx$
3. $x^3 - 2x^2 + 4x - 8$
4. $y^3 - 2y^2 + 5y - 10$
5. $2a - 6 - ab^2 + 3b^2$
6. $x^3 + 3x^2 - 9x - 27$
7. $x^2 - 2x + 1 - y^2$
8. $xy^3 + 2y^2 - xy - 2$
9. $4x^2 - y^2 + 4y - 4$
10. $x^6 - 7x^3 - 8$
11. $x^2 + 2xy + y^2 - z^2 + 2zw - w^2$
12. $4a^2 - x^2 + b^2 - y^2 - 4ab - 2xy$
13. $x^2 + 4xy + 4y^2 - x - 2y - 6$
14. $x^3 - 5x^2 - x + 5$
15. $x^4 - 7x^2y^2 + 9y^4$
16. $y^4 + y^2 + 25$
17. $a^4 + 2a^2b^2 + 9b^4$
18. $x^4 + 5x^2 + 9$
19. $b^4 + 6b^2c^2 + 25c^2$
20. $25x^2 + 30xy + 9y^2 + 15x + 9y + 2$
21. $3ax - 6ay + 4bx - 8by + cx - 2cy$
22. $20xy + 7zw - 5yz - 28xw$
23. $z^4 + 4z^3 - 2z - 8$
24. $x^4 + 4y^4$
25. $a^8 - b^8$
26. $x^6 + 1$
27. $x^2 + 2xy - z^2 - 2yz$
28. $(x^2 + 2x - 3)^2 - 4$
29. $(x - y - 2z)^2 - (2x + y - z)^2$
30. $2(x + 2)^2(x - 3) + 3(x + 2)(x - 3)^2$

3-6 SIMPLIFICATION OF FRACTIONS

A basic principle for fractions, algebraic as well as arithmetic, states that the value of a fraction is not changed if its numerator and denominator are both multiplied or both divided by the same quantity (not zero). This principle was stated in Theorem 2-8. Hence, the simplification or reduction of a fraction to lowest terms is always possible. Factor both the numerator and denominator into their prime factors and, by using the basic principle, divide the numerator and denominator by the product of all their common factors.

Example 1. Reduce $(8x^4y^7)/(12x^6y^3)$ to lowest terms.

$$\text{Solution} \quad \frac{8x^4y^7}{12x^6y^3} \equiv \frac{2^3x^4y^7}{2^2 \cdot 3x^6y^3} \equiv \frac{2^2x^4y^3 \cdot 2y^4}{2^2x^4y^3 \cdot 3x^2}$$

By dividing both numerator and denominator by $2^2x^4y^3$, we have

$$\frac{8x^4y^7}{12x^6y^3} \equiv \frac{2y^4}{3x^2}$$

3-6 SIMPLIFICATION OF FRACTIONS

Example 2. Reduce $(x^2 - 7x + 10)/(2x^2 - x - 6)$ to lowest terms.

Solution. If we factor both numerator and denominator, we have

$$\frac{x^2 - 7x + 10}{2x^2 - x - 6} \equiv \frac{(x - 5)(x - 2)}{(2x + 3)(x - 2)},$$

and dividing both numerator and denominator by $x - 2$, that is, applying Theorem 2-8, we get

$$\frac{x^2 - 7x + 10}{2x^2 - x - 6} \equiv \frac{x - 5}{2x + 3}.$$

The elimination of a common factor by dividing the numerator and denominator of a fraction by this factor is called *multiplicative cancellation*. Such a process should be done with care, for Theorem 2-8 is true only when $x \neq 0$. In this case the identity is true for all values of x except $x = 2$ or $x = -\frac{3}{2}$, which are not permissible values.

Example 3. Reduce $(12x^2 + 30x - 72)/(52x - 8x^2 - 60)$ to lowest terms.

$$\text{Solution} \quad \frac{12x^2 + 30x - 72}{52x - 8x^2 - 60} \equiv \frac{6(2x - 3)(x + 4)}{4(3 - 2x)(x - 5)} \equiv \frac{3(x + 4)}{2(5 - x)}.$$

This identity follows from the fact that $2x - 3 = -(3 - 2x)$. (Recall Problem 4, Article 2-4.)

PROBLEMS

Reduce the following to lowest terms.

- | | |
|---|---|
| 1. $\frac{28}{63}$ | 2. $\frac{27x^3}{225x^5}$ |
| 3. $\frac{a^4 x^3 y}{a^2 x y^3}$ | 4. $\frac{a^2 + ab}{3a + 2a^3}$ |
| 5. $\frac{a^2 x - a^2 y}{ax^2 - ay^2}$ | 6. $\frac{24a^2}{6a^2 - 9a}$ |
| 7. $\frac{x^2 - 1}{x^2 - x}$ | 8. $\frac{x^2 - 4x + 4}{x^2 - 4}$ |
| 9. $\frac{x^2 - 16}{x^2 - 8x + 16}$ | 10. $\frac{a^2 - 3a - 4}{a^2 - a - 12}$ |
| 11. $\frac{y^2 - y - 6}{y^2 + 2y - 15}$ | 12. $\frac{2x^2 + 5x - 12}{4x^2 - 4x - 3}$ |
| 13. $\frac{6a^2 - 7a - 3}{4a^2 - 8a + 3}$ | 14. $\frac{ax + ay - bx - by}{am - bm - an + bn}$ |

EXTENSIONS OF THE LOGIC OF ALGEBRA 3-7

15. $\frac{14x - 24 - 2x^2}{x^2 + x - 20}$

17. $\frac{x^2 - 36}{x^3 - 216}$

19. $\frac{2(x^2 - y^2)xy + x^4 - y^4}{x^4 - y^4}$

21. $\frac{4a^2 - 1}{12a^2 + a - 4a^3 - 3}$

23. $\frac{(x^2 - 16)(x^2 - 4x + 16)}{x^3 + 64}$

16. $\frac{(4x^2 - 9y^2)(18x - 12)}{(2x - 3y)(12x - 8)}$

18. $\frac{2x^2 - 14x + 20}{7x - 2x^2 - 6}$

20. $\frac{y^6 + 64}{y^4 - 4y^2 + 16}$

22. $\frac{a^2 - 2ab + 3b^2}{a^4 + 2a^2b^2 + 9b^4}$

24. $\frac{15ab - 20a - 21b + 28}{21 - a - 10a^2}$

3-7 ADDITION OF FRACTIONS

The algebraic sum of two or more fractions having the same denominator is a fraction with the common denominator and a numerator which is the algebraic sum of the numerators of the fractions considered. This was proved in Problem 13, Article 2-4.

Illustration. $\frac{2x^2}{x-4} - \frac{3x}{x-4} + \frac{5}{x-4} \equiv \frac{2x^2 - 3x + 5}{x-4}$.

To find the algebraic sum of two or more fractions with different denominators, we must replace the fractions with equivalent fractions having the same denominators. It is preferable to use the *least common denominator* (LCD). The LCD of two or more fractions consists of the product of all the unique prime factors in the denominators, each with an exponent equal to the largest exponent with which the factor appears, and is really a result of the following important theorem.

Theorem 3-5. $\frac{a}{b} + \frac{c}{d} \equiv \frac{ad + bc}{bd}$ ($b, d \neq 0$).

Proof. We have

$$\frac{a}{b} + \frac{c}{d} \equiv \frac{ad}{bd} + \frac{bc}{bd},$$

by Theorem 2-8. If now use Problem 13, Article 2-4, we have

$$\frac{ad}{bd} + \frac{bc}{bd} \equiv \frac{ad + bc}{bd},$$

which is our required result.

Example 1. Find the LCD of the fractions

$$\frac{3x}{x^2 - 4x + 4}, \quad \frac{5x^2}{3(x^2 - 4)}, \quad \frac{2}{2x^2 - x - 6}.$$

3-7 ADDITION OF FRACTIONS

Solution. Factoring each denominator, we have

$$\begin{aligned}x^2 - 4x + 4 &\equiv (x - 2)^2, \\3(x^2 - 4) &\equiv 3(x + 2)(x - 2), \\2x^2 - x - 6 &\equiv (2x + 3)(x - 2).\end{aligned}$$

The LCD is $3(x + 2)(x - 2)^2(2x + 3)$.

After the LCD has been determined, equivalent fractions may be formed. Divide the LCD of a given fraction by the denominator of that fraction, and then multiply both numerator and denominator of the given fraction by the result. The equivalent fractions may now be added, as in the illustration above.

Example 2. Change the following fractions to equivalent ones, with their LCD as denominator, and find their sum.

$$\frac{4}{x + 2}, \quad \frac{x + 3}{x^2 - 4}, \quad \frac{2x + 1}{x - 2}.$$

Solution. The LCD is $(x + 2)(x - 2)$. Therefore,

$$\begin{aligned}\frac{4}{x + 2} &\equiv \frac{4(x - 2)}{(x + 2)(x - 2)}, & \frac{x + 3}{x^2 - 4} &\equiv \frac{x + 3}{(x + 2)(x - 2)}, \\ \frac{2x + 1}{x - 2} &\equiv \frac{(2x + 1)(x + 2)}{(x + 2)(x - 2)},\end{aligned}$$

and

$$\begin{aligned}\frac{4}{x + 2} + \frac{x + 3}{x^2 - 4} + \frac{2x + 1}{x - 2} & \\ &\equiv \frac{4(x - 2)}{(x + 2)(x - 2)} + \frac{x + 3}{(x + 2)(x - 2)} + \frac{(2x + 1)(x + 2)}{(x + 2)(x - 2)} \\ &\equiv \frac{(4x - 8) + (x + 3) + (2x^2 + 5x + 2)}{x^2 - 4} \\ &\equiv \frac{2x^2 + 10x - 3}{x^2 - 4}.\end{aligned}$$

PROBLEMS

Reduce the following to single fractions and simplify.

1. $\frac{2}{3} + \frac{5}{8} - \frac{3}{10}$

2. $5 - \frac{4}{9} - \frac{7}{15}$

3. $\frac{3x}{4y} - \frac{4y}{3x}$

4. $\frac{a^2}{b} - \frac{b^2}{a}$

5. $\frac{2x + 3}{6} - \frac{4x - 7}{9}$

6. $\frac{3x - 1}{5} + \frac{4 - 5x}{6}$

EXTENSIONS OF THE LOGIC OF ALGEBRA 3-8

7. $x + y + \frac{x^2}{x - y}$

9. $\frac{3x - 2y}{5x - 3} + \frac{2x - y}{3 - 5x}$

11. $\frac{5}{x} - \frac{4}{y} + \frac{3}{z}$

13. $\frac{2x - 1}{4 - x} + \frac{x + 2}{3x - 12}$

15. $\frac{x - 1}{2x^2 - 13x + 15} + \frac{x + 3}{2x^2 - 15x + 18}$

17. $\frac{3}{a - 3} + \frac{a^2 + 2}{a^3 - 27}$

19. $\frac{2}{x^2 + 3x + 2} - \frac{3}{x^2 + 5x + 6} - \frac{4}{x^2 + 4x + 3}$

20. $x + 6 + \frac{5x + 1}{12x^2 + 5x - 2} - \frac{x}{3x + 2}$

21. $2y - 3 + \frac{y - 2}{4y^2 - 12y + 9} + \frac{y + 2}{2y^2 - y - 3}$

22. $\frac{1}{(x - y)(y - z)} + \frac{1}{(y - z)(z - x)} + \frac{1}{(z - x)(x - y)}$

23. $\frac{x}{(x - y)(y - z)} + \frac{y}{(y - z)(z - x)} + \frac{z}{(z - x)(x - y)}$

24. $\frac{2x - 1}{2x^2 - x - 6} + \frac{x + 3}{6x^2 + x - 12} - \frac{2x - 3}{3x^2 - 10x + 8}$

8. $\frac{x + 1}{x + 2} - \frac{x + 3}{x}$

10. $\frac{2}{12x^2 - 3} + \frac{3}{2x - 4x^2}$

12. $\frac{4}{x^2 - 4x - 5} + \frac{2}{x^2 - 1}$

14. $\frac{x + 5}{x^2 + 7x + 10} - \frac{x - 1}{x^2 + 5x + 6}$

16. $\frac{2x + 3}{3x^2 + x - 2} - \frac{3x - 4}{2x^2 - 3x - 5}$

18. $\frac{2xy}{x^3 + y^3} - \frac{x}{x^2 - xy + y^2}$

3-8 MULTIPLICATION AND DIVISION OF FRACTIONS

In algebra, as in arithmetic, the product of two or more fractions is a fraction whose numerator is the product of all numerators and whose denominator is the product of all denominators. In obtaining these products, the process of dividing out factors common to the numerator and denominator may be used. Results should be reduced to their simplest form. Note in Illustration 1 that this method is a direct application of Theorem 2-7, followed by an application of Theorem 2-8.

$$\begin{aligned} \text{Illustration 1. } \frac{x - 4}{2x + 8} \cdot \frac{4x + 8}{x^2 - 16} &\equiv \frac{(x - 4) \cdot 2^2 \cdot (x + 2)}{2(x + 4)(x + 4)(x - 4)} \\ &\equiv \frac{2(x + 2)}{(x + 4)^2} \end{aligned}$$

Algebra Sample in Nemeth Code

Nemeth Symbols			
⠆⠆	punctuation indicator	⠆⠆	minus sign
⠆⠆	begin superscript	⠆⠆⠆⠆	plus or minus sign
⠆⠆	return to baseline after superscript	⠆⠆⠆⠆	minus or plus sign
⠆⠆	begin fraction	⠆⠆	times sign (dot)
⠆⠆	horizontal fraction line	⠆⠆⠆⠆	slash
⠆⠆	end fraction	⠆⠆⠆⠆	equivalent sign (3 horizontal bars)
⠆⠆	opening parenthesis	⠆⠆⠆⠆	equal sign
⠆⠆	closing parenthesis	⠆⠆⠆⠆⠆⠆	not equal sign
⠆⠆⠆⠆	opening bracket	⠆⠆⠆⠆	asterisk
⠆⠆⠆⠆	closing bracket	⠆⠆	mathematical comma
⠆⠆	plus sign		

Algebra Sample in UEBC

UEBC Symbols			
⠠	grade one symbol	⠨	superscript next item
⠠⠠⠠	begin grade one passage	⠨	begin compound item
⠠	(before a space)	⠨	end compound item
⠠⠠⠠⠠⠠	end grade one passage	⠨⠨	opening parenthesis
⠠⠠⠠⠠⠠	(on a line by itself)	⠨⠨	closing parenthesis
⠠⠠⠠⠠⠠	begin grade one passage	⠨⠨	opening bracket
⠠⠠⠠⠠	(on a line by itself)	⠨⠨	closing bracket
⠠⠠⠠	end grade one passage	⠨	decimal point
⠠⠠⠠	begin capitalized passage	⠨⠨	plus sign
⠠	(before a space)	⠨⠨	minus sign
⠠	end capitalized passage	⠨⠨	plus or minus sign
⠠⠠	italic word	⠨⠨	minus or plus sign
⠠⠠	begin italic passage	⠨⠨	times sign (dot)
⠠	(before a space)	⠨⠨	equal sign
⠠	end italic passage	⠨⠨⠠⠠	not equal sign
⠠⠠	bold word	⠨⠨	equivalent sign (3 horizontal bars)
⠠⠠	begin bold passage	⠨⠨	slash
⠠	(before a space)	⠨⠨	asterisk
⠠	end bold passage		
⠠	begin fraction		
⠠⠠	fraction line		
⠠	end fraction		

1. $2x^2 + 3x - 5$ $4x^2 - 7x + 1$ $5x^2 + 2x - 3$ $6x^2 - 4x + 8$

2. $3x^2 + 2x - 1$ $4x^2 - 5x + 2$ $5x^2 + 3x - 4$ $6x^2 - 2x + 7$ $7x^2 + 4x - 5$ $8x^2 - 3x + 6$ $9x^2 + 2x - 3$ $10x^2 - 4x + 8$ $11x^2 + 5x - 6$ $12x^2 - 6x + 9$ $13x^2 + 7x - 8$ $14x^2 - 8x + 10$ $15x^2 + 9x - 11$ $16x^2 - 10x + 12$ $17x^2 + 11x - 13$ $18x^2 - 12x + 14$ $19x^2 + 13x - 15$ $20x^2 - 14x + 16$ $21x^2 + 15x - 17$ $22x^2 - 16x + 18$ $23x^2 + 17x - 19$ $24x^2 - 18x + 20$ $25x^2 + 19x - 21$ $26x^2 - 20x + 22$ $27x^2 + 21x - 23$ $28x^2 - 22x + 24$ $29x^2 + 23x - 25$ $30x^2 - 24x + 26$ $31x^2 + 25x - 27$ $32x^2 - 26x + 28$ $33x^2 + 27x - 29$ $34x^2 - 28x + 30$ $35x^2 + 29x - 31$ $36x^2 - 30x + 32$ $37x^2 + 31x - 33$ $38x^2 - 32x + 34$ $39x^2 + 33x - 35$ $40x^2 - 34x + 36$ $41x^2 + 35x - 37$ $42x^2 - 36x + 38$ $43x^2 + 37x - 39$ $44x^2 - 38x + 40$ $45x^2 + 39x - 41$ $46x^2 - 40x + 42$ $47x^2 + 41x - 43$ $48x^2 - 42x + 44$ $49x^2 + 43x - 45$ $50x^2 - 44x + 46$ $51x^2 + 45x - 47$ $52x^2 - 46x + 48$ $53x^2 + 47x - 49$ $54x^2 - 48x + 50$ $55x^2 + 49x - 51$ $56x^2 - 50x + 52$ $57x^2 + 51x - 53$ $58x^2 - 52x + 54$ $59x^2 + 53x - 55$ $60x^2 - 54x + 56$ $61x^2 + 55x - 57$ $62x^2 - 56x + 58$ $63x^2 + 57x - 59$ $64x^2 - 58x + 60$ $65x^2 + 59x - 61$ $66x^2 - 60x + 62$ $67x^2 + 61x - 63$ $68x^2 - 62x + 64$ $69x^2 + 63x - 65$ $70x^2 - 64x + 66$ $71x^2 + 65x - 67$ $72x^2 - 66x + 68$ $73x^2 + 67x - 69$ $74x^2 - 68x + 70$ $75x^2 + 69x - 71$ $76x^2 - 70x + 72$ $77x^2 + 71x - 73$ $78x^2 - 72x + 74$ $79x^2 + 73x - 75$ $80x^2 - 74x + 76$ $81x^2 + 75x - 77$ $82x^2 - 76x + 78$ $83x^2 + 77x - 79$ $84x^2 - 78x + 80$ $85x^2 + 79x - 81$ $86x^2 - 80x + 82$ $87x^2 + 81x - 83$ $88x^2 - 82x + 84$ $89x^2 + 83x - 85$ $90x^2 - 84x + 86$ $91x^2 + 85x - 87$ $92x^2 - 86x + 88$ $93x^2 + 87x - 89$ $94x^2 - 88x + 90$ $95x^2 + 89x - 91$ $96x^2 - 90x + 92$ $97x^2 + 91x - 93$ $98x^2 - 92x + 94$ $99x^2 + 93x - 95$ $100x^2 - 94x + 96$

3. $2x^2 + 3x - 5$ $4x^2 - 7x + 1$ $5x^2 + 2x - 3$ $6x^2 - 4x + 8$ $7x^2 + 4x - 5$ $8x^2 - 3x + 6$ $9x^2 + 2x - 3$ $10x^2 - 4x + 8$ $11x^2 + 5x - 6$ $12x^2 - 6x + 9$ $13x^2 + 7x - 8$ $14x^2 - 8x + 10$ $15x^2 + 9x - 11$ $16x^2 - 10x + 12$ $17x^2 + 11x - 13$ $18x^2 - 12x + 14$ $19x^2 + 13x - 15$ $20x^2 - 14x + 16$ $21x^2 + 15x - 17$ $22x^2 - 16x + 18$ $23x^2 + 17x - 19$ $24x^2 - 18x + 20$ $25x^2 + 19x - 21$ $26x^2 - 20x + 22$ $27x^2 + 21x - 23$ $28x^2 - 22x + 24$ $29x^2 + 23x - 25$ $30x^2 - 24x + 26$ $31x^2 + 25x - 27$ $32x^2 - 26x + 28$ $33x^2 + 27x - 29$ $34x^2 - 28x + 30$ $35x^2 + 29x - 31$ $36x^2 - 30x + 32$ $37x^2 + 31x - 33$ $38x^2 - 32x + 34$ $39x^2 + 33x - 35$ $40x^2 - 34x + 36$ $41x^2 + 35x - 37$ $42x^2 - 36x + 38$ $43x^2 + 37x - 39$ $44x^2 - 38x + 40$ $45x^2 + 39x - 41$ $46x^2 - 40x + 42$ $47x^2 + 41x - 43$ $48x^2 - 42x + 44$ $49x^2 + 43x - 45$ $50x^2 - 44x + 46$ $51x^2 + 45x - 47$ $52x^2 - 46x + 48$ $53x^2 + 47x - 49$ $54x^2 - 48x + 50$ $55x^2 + 49x - 51$ $56x^2 - 50x + 52$ $57x^2 + 51x - 53$ $58x^2 - 52x + 54$ $59x^2 + 53x - 55$ $60x^2 - 54x + 56$ $61x^2 + 55x - 57$ $62x^2 - 56x + 58$ $63x^2 + 57x - 59$ $64x^2 - 58x + 60$ $65x^2 + 59x - 61$ $66x^2 - 60x + 62$ $67x^2 + 61x - 63$ $68x^2 - 62x + 64$ $69x^2 + 63x - 65$ $70x^2 - 64x + 66$ $71x^2 + 65x - 67$ $72x^2 - 66x + 68$ $73x^2 + 67x - 69$ $74x^2 - 68x + 70$ $75x^2 + 69x - 71$ $76x^2 - 70x + 72$ $77x^2 + 71x - 73$ $78x^2 - 72x + 74$ $79x^2 + 73x - 75$ $80x^2 - 74x + 76$ $81x^2 + 75x - 77$ $82x^2 - 76x + 78$ $83x^2 + 77x - 79$ $84x^2 - 78x + 80$ $85x^2 + 79x - 81$ $86x^2 - 80x + 82$ $87x^2 + 81x - 83$ $88x^2 - 82x + 84$ $89x^2 + 83x - 85$ $90x^2 - 84x + 86$ $91x^2 + 85x - 87$ $92x^2 - 86x + 88$ $93x^2 + 87x - 89$ $94x^2 - 88x + 90$ $95x^2 + 89x - 91$ $96x^2 - 90x + 92$ $97x^2 + 91x - 93$ $98x^2 - 92x + 94$ $99x^2 + 93x - 95$ $100x^2 - 94x + 96$

$$3x^2 + 2x - 5 = (x - 2)(3x + 7)$$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$x^2 - 4x + 4 = (x - 2)^2$$

$$x^2 - 9 = (x - 3)(x + 3)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 - 10x + 25 = (x - 5)^2$$

$$x^2 + 11x + 28 = (x + 4)(x + 7)$$

$$x^2 - 16 = (x - 4)(x + 4)$$

$$x^2 + 13x + 40 = (x + 5)(x + 8)$$

$$x^2 - 25 = (x - 5)(x + 5)$$

$$x^2 + 15x + 50 = (x + 5)(x + 10)$$

$$x^2 - 1 = (x - 1)(x + 1)$$

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

$$x^2 - 36 = (x - 6)(x + 6)$$

$$x^2 + 9x + 14 = (x + 2)(x + 7)$$

$$x^2 - 49 = (x - 7)(x + 7)$$

$$x^2 + 17x + 52 = (x + 4)(x + 13)$$

$$x^2 - 81 = (x - 9)(x + 9)$$

$$x^2 + 6x + 8 = (x + 2)(x + 4)$$

$$x^2 - 100 = (x - 10)(x + 10)$$

$$x^2 + 12x + 35 = (x + 5)(x + 7)$$

$$x^2 - 64 = (x - 8)(x + 8)$$

$$x^2 + 19x + 70 = (x + 5)(x + 14)$$

$$x^2 - 81 = (x - 9)(x + 9)$$

1. $(x^2 + 3x + 2)(x^2 + 5x + 6)$

2. $(x^2 + 4x + 4)(x^2 + 7x + 10)$

3. $(x^2 + 6x + 9)(x^2 + 8x + 15)$

4. $(x^2 + 5x + 6)(x^2 + 9x + 14)$

5. $(x^2 + 7x + 12)(x^2 + 10x + 24)$

6. $(x^2 + 8x + 16)(x^2 + 11x + 30)$

7. $(x^2 + 9x + 20)(x^2 + 12x + 36)$

8. $(x^2 + 10x + 25)(x^2 + 13x + 42)$

9. $(x^2 + 11x + 30)(x^2 + 14x + 49)$

10. $(x^2 + 12x + 36)(x^2 + 15x + 56)$

11. $(x^2 + 13x + 42)(x^2 + 16x + 63)$

12. $(x^2 + 14x + 49)(x^2 + 17x + 72)$

13. $(x^2 + 15x + 50)(x^2 + 18x + 81)$

14. $(x^2 + 16x + 56)(x^2 + 19x + 90)$

15. $(x^2 + 17x + 63)(x^2 + 20x + 100)$

16. $(x^2 + 18x + 72)(x^2 + 21x + 112)$

17. $(x^2 + 19x + 81)(x^2 + 22x + 126)$

18. $(x^2 + 20x + 90)(x^2 + 23x + 144)$

19. $(x^2 + 21x + 100)(x^2 + 24x + 162)$

20. $(x^2 + 22x + 112)(x^2 + 25x + 182)$

$(x + 3)(x - 5) = x^2 - 5x + 3x - 15 = x^2 - 2x - 15$

$(x^2 + 2x - 15) + (-x^2 - 7x + 15) = x^2 + 2x - 15 - x^2 - 7x + 15 = -5x$

$(-5x) + (3x + 10) = -5x + 3x + 10 = -2x + 10$

$(-2x + 10) + (x + 5) = -2x + 10 + x + 5 = -x + 15$

$(-x + 15) + (x - 3) = -x + 15 + x - 3 = 12$

$(12) + (x - 5) = 12 + x - 5 = x + 7$

$(x + 7) + (-x - 3) = x + 7 - x - 3 = 4$

$(4) + (x - 5) = 4 + x - 5 = x - 1$

$(x - 1) + (-x - 3) = x - 1 - x - 3 = -4$

$(-4) + (x - 5) = -4 + x - 5 = x - 9$

$(x - 9) + (-x - 3) = x - 9 - x - 3 = -12$

$(-12) + (x - 5) = -12 + x - 5 = x - 17$

$(x - 17) + (-x - 3) = x - 17 - x - 3 = -20$

$(-20) + (x - 5) = -20 + x - 5 = x - 25$

$(x - 25) + (-x - 3) = x - 25 - x - 3 = -28$

$(-28) + (x - 5) = -28 + x - 5 = x - 33$

$(x - 33) + (-x - 3) = x - 33 - x - 3 = -36$

$(-36) + (x - 5) = -36 + x - 5 = x - 41$

$(x - 41) + (-x - 3) = x - 41 - x - 3 = -44$

$(-44) + (x - 5) = -44 + x - 5 = x - 49$

$(x - 49) + (-x - 3) = x - 49 - x - 3 = -52$

$(-52) + (x - 5) = -52 + x - 5 = x - 57$

$(x - 57) + (-x - 3) = x - 57 - x - 3 = -60$

$(-60) + (x - 5) = -60 + x - 5 = x - 65$

$(x - 65) + (-x - 3) = x - 65 - x - 3 = -68$

$(-68) + (x - 5) = -68 + x - 5 = x - 73$

$(x - 73) + (-x - 3) = x - 73 - x - 3 = -76$

$(-76) + (x - 5) = -76 + x - 5 = x - 81$

$(x - 81) + (-x - 3) = x - 81 - x - 3 = -84$

$(-84) + (x - 5) = -84 + x - 5 = x - 89$

$(x - 89) + (-x - 3) = x - 89 - x - 3 = -92$

$(-92) + (x - 5) = -92 + x - 5 = x - 97$

$(x - 97) + (-x - 3) = x - 97 - x - 3 = -100$

$5x^2 + 7x - 3 = 5x^2 + 10x - 3 - 3x = (5x^2 + 10x - 3) - 3x$

$5x^2 + 10x - 3 = (5x + 3)(x - 1) + 6$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x$

$5x^2 + 7x - 3 = (5x + 3)(x - 1) + 6 - 3x + 3x + 9 - 9 + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x + 3x$

$$x^2 + 2x + 1 = (x + 1)^2$$

$$x^2 - 4 = (x - 2)(x + 2)$$

$$x^2 + 5x + 6 = (x + 2)(x + 3)$$

$$x^2 - 9 = (x - 3)(x + 3)$$

$$x^2 + 7x + 12 = (x + 3)(x + 4)$$

$$x^2 - 16 = (x - 4)(x + 4)$$

$$x^2 + 8x + 15 = (x + 3)(x + 5)$$

$$x^2 - 25 = (x - 5)(x + 5)$$

$$x^2 + 9x + 14 = (x + 2)(x + 7)$$

$$x^2 + 11x + 28 = (x + 4)(x + 7)$$

$$x^2 - 10 = (x - \sqrt{10})(x + \sqrt{10})$$

$$x^2 + 13x + 42 = (x + 6)(x + 7)$$

$$x^2 - 12 = (x - 2\sqrt{3})(x + 2\sqrt{3})$$

$$x^2 + 14x + 49 = (x + 7)^2$$

$$x^2 - 18 = (x - 3\sqrt{2})(x + 3\sqrt{2})$$

$$x^2 + 15x + 50 = (x + 5)(x + 10)$$

$$x^2 - 20 = (x - 2\sqrt{5})(x + 2\sqrt{5})$$

$$x^2 + 16x + 64 = (x + 8)^2$$

$$x^2 - 24 = (x - 2\sqrt{6})(x + 2\sqrt{6})$$

$$x^2 + 17x + 72 = (x + 8)(x + 9)$$

1. $2x^2 + 3x - 4$

2. $5x^2 - 2x + 7$

3. $3x^2 + 4x - 1$

4. $2x^2 + 3x - 4$

5. $5x^2 - 2x + 7$

6. $3x^2 + 4x - 1$

7. $2x^2 + 3x - 4$

8. $5x^2 - 2x + 7$

9. $3x^2 + 4x - 1$

10. $2x^2 + 3x - 4$

11. $5x^2 - 2x + 7$

12. $3x^2 + 4x - 1$

13. $2x^2 + 3x - 4$

14. $5x^2 - 2x + 7$

15. $3x^2 + 4x - 1$

16. $2x^2 + 3x - 4$

17. $5x^2 - 2x + 7$

18. $3x^2 + 4x - 1$

19. $2x^2 + 3x - 4$

20. $5x^2 - 2x + 7$

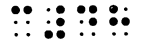
21. $3x^2 + 4x - 1$

22. $2x^2 + 3x - 4$

23. $5x^2 - 2x + 7$

24. $3x^2 + 4x - 1$

25. $2x^2 + 3x - 4$



Braille text consisting of multiple lines of mathematical expressions and symbols, including numbers and algebraic terms.

Braille text consisting of multiple lines of mathematical expressions and symbols, including numbers and algebraic terms.

18. $(2x + 3)^2 - (x - 4)^2$

19. $\frac{x^2 - 4}{x^2 + 5x + 4} \cdot \frac{x^2 + 2x - 8}{x^2 - 3x - 4}$

20. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

21. $\frac{x^2 + 5x + 4}{x^2 - 3x - 4} + \frac{x^2 + 2x - 8}{x^2 - 3x - 4}$

22. $\frac{x^2 + 5x + 4}{x^2 - 3x - 4} - \frac{x^2 + 2x - 8}{x^2 - 3x - 4}$

23. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

24. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

25. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

26. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

27. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

28. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

29. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

30. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

31. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

32. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

33. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

34. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

35. $\frac{1}{x^2 + 3x - 4} - \frac{1}{x^2 + 4x + 4}$

1. $2x^2 + 3x - 5$ $4x^2 - 7x + 1$ $5x^2 + 2x - 3$

2. $3x^2 + 4x - 6$ $2x^2 - 5x + 8$ $7x^2 + 1x - 4$

3. $6x^2 + 8x - 9$ $1x^2 - 3x + 2$ $4x^2 + 5x - 1$

4. $8x^2 + 9x - 7$ $3x^2 - 6x + 5$ $2x^2 + 4x - 3$

5. $9x^2 + 10x - 8$ $5x^2 - 8x + 6$ $1x^2 + 3x - 2$

6. $7x^2 + 6x - 5$ $4x^2 - 9x + 7$ $3x^2 + 7x - 4$

7. $5x^2 + 4x - 3$ $2x^2 - 6x + 4$ $6x^2 + 5x - 2$

8. $3x^2 + 2x - 1$ $8x^2 - 7x + 3$ $1x^2 + 2x - 1$

9. $4x^2 + 3x - 2$ $7x^2 - 5x + 1$ $2x^2 + 1x - 1$

10. $6x^2 + 5x - 4$ $3x^2 - 4x + 2$ $5x^2 + 6x - 3$

11. $8x^2 + 7x - 6$ $1x^2 - 2x + 1$ $4x^2 + 3x - 2$

12. $2x^2 + 1x - 1$ $9x^2 - 8x + 4$ $3x^2 + 4x - 3$

13. $5x^2 + 4x - 3$ $6x^2 - 5x + 2$ $1x^2 + 2x - 1$

14. $3x^2 + 2x - 1$ $4x^2 - 3x + 1$ $7x^2 + 8x - 5$

15. $2x^2 + 1x - 1$ $8x^2 - 7x + 3$ $1x^2 + 2x - 1$

16. $4x^2 + 3x - 2$ $7x^2 - 5x + 1$ $2x^2 + 1x - 1$

17. $6x^2 + 5x - 4$ $3x^2 - 4x + 2$ $5x^2 + 6x - 3$

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3x^2 + 2x - 5 + 4x^2 + 3x - 2

7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2 = 7x^2 + 5x - 7

3x^2 + 2x - 5 + 4x^2 + 3x - 2

1. $(x^2 + 3x + 2)(x^2 + 5x + 6)$

2. $(x^2 + 4x + 4)(x^2 + 6x + 9)$

3. $(x^2 + 7x + 12)(x^2 + 8x + 15)$

4. $(x^2 + 9x + 14)(x^2 + 10x + 21)$

5. $(x^2 + 11x + 30)(x^2 + 12x + 36)$

6. $(x^2 + 13x + 42)(x^2 + 14x + 49)$

7. $(x^2 + 15x + 50)(x^2 + 16x + 64)$

8. $(x^2 + 17x + 60)(x^2 + 18x + 81)$

9. $(x^2 + 19x + 70)(x^2 + 20x + 100)$

10. $(x^2 + 21x + 80)(x^2 + 22x + 121)$

11. $(x^2 + 23x + 90)(x^2 + 24x + 144)$

12. $(x^2 + 25x + 100)(x^2 + 26x + 169)$

13. $(x^2 + 27x + 110)(x^2 + 28x + 196)$

14. $(x^2 + 29x + 120)(x^2 + 30x + 225)$

15. $(x^2 + 31x + 130)(x^2 + 32x + 256)$

16. $(x^2 + 33x + 140)(x^2 + 34x + 289)$

17. $(x^2 + 35x + 150)(x^2 + 36x + 324)$

18. $(x^2 + 37x + 160)(x^2 + 38x + 361)$

19. $(x^2 + 39x + 170)(x^2 + 40x + 400)$

20. $(x^2 + 41x + 180)(x^2 + 42x + 441)$

21. $(x^2 + 43x + 190)(x^2 + 44x + 484)$

22. $(x^2 + 45x + 200)(x^2 + 46x + 529)$

23. $(x^2 + 47x + 210)(x^2 + 48x + 576)$

24. $(x^2 + 49x + 220)(x^2 + 50x + 625)$

25. $(x^2 + 51x + 230)(x^2 + 52x + 676)$

26. $(x^2 + 53x + 240)(x^2 + 54x + 729)$

27. $(x^2 + 55x + 250)(x^2 + 56x + 784)$

28. $(x^2 + 57x + 260)(x^2 + 58x + 841)$

29. $(x^2 + 59x + 270)(x^2 + 60x + 900)$

30. $(x^2 + 61x + 280)(x^2 + 62x + 961)$

31. $(x^2 + 63x + 290)(x^2 + 64x + 1024)$

32. $(x^2 + 65x + 300)(x^2 + 66x + 1089)$

33. $(x^2 + 67x + 310)(x^2 + 68x + 1156)$

34. $(x^2 + 69x + 320)(x^2 + 70x + 1225)$

35. $(x^2 + 71x + 330)(x^2 + 72x + 1296)$

36. $(x^2 + 73x + 340)(x^2 + 74x + 1369)$

37. $(x^2 + 75x + 350)(x^2 + 76x + 1444)$

38. $(x^2 + 77x + 360)(x^2 + 78x + 1521)$

39. $(x^2 + 79x + 370)(x^2 + 80x + 1600)$

40. $(x^2 + 81x + 380)(x^2 + 82x + 1681)$

41. $(x^2 + 83x + 390)(x^2 + 84x + 1764)$

42. $(x^2 + 85x + 400)(x^2 + 86x + 1849)$

43. $(x^2 + 87x + 410)(x^2 + 88x + 1936)$

44. $(x^2 + 89x + 420)(x^2 + 90x + 2025)$

45. $(x^2 + 91x + 430)(x^2 + 92x + 2116)$

46. $(x^2 + 93x + 440)(x^2 + 94x + 2209)$

47. $(x^2 + 95x + 450)(x^2 + 96x + 2304)$

48. $(x^2 + 97x + 460)(x^2 + 98x + 2401)$

49. $(x^2 + 99x + 470)(x^2 + 100x + 2500)$

50. $(x^2 + 101x + 480)(x^2 + 102x + 2601)$

51. $(x^2 + 103x + 490)(x^2 + 104x + 2704)$

52. $(x^2 + 105x + 500)(x^2 + 106x + 2809)$

53. $(x^2 + 107x + 510)(x^2 + 108x + 2916)$

54. $(x^2 + 109x + 520)(x^2 + 110x + 3025)$

55. $(x^2 + 111x + 530)(x^2 + 112x + 3136)$

56. $(x^2 + 113x + 540)(x^2 + 114x + 3249)$

57. $(x^2 + 115x + 550)(x^2 + 116x + 3364)$

58. $(x^2 + 117x + 560)(x^2 + 118x + 3481)$

59. $(x^2 + 119x + 570)(x^2 + 120x + 3600)$

60. $(x^2 + 121x + 580)(x^2 + 122x + 3721)$

61. $(x^2 + 123x + 590)(x^2 + 124x + 3844)$

62. $(x^2 + 125x + 600)(x^2 + 126x + 3969)$

63. $(x^2 + 127x + 610)(x^2 + 128x + 4096)$

64. $(x^2 + 129x + 620)(x^2 + 130x + 4225)$

65. $(x^2 + 131x + 630)(x^2 + 132x + 4356)$

66. $(x^2 + 133x + 640)(x^2 + 134x + 4489)$

67. $(x^2 + 135x + 650)(x^2 + 136x + 4624)$

68. $(x^2 + 137x + 660)(x^2 + 138x + 4761)$

69. $(x^2 + 139x + 670)(x^2 + 140x + 4900)$

70. $(x^2 + 141x + 680)(x^2 + 142x + 5041)$

71. $(x^2 + 143x + 690)(x^2 + 144x + 5184)$

72. $(x^2 + 145x + 700)(x^2 + 146x + 5329)$

73. $(x^2 + 147x + 710)(x^2 + 148x + 5476)$

74. $(x^2 + 149x + 720)(x^2 + 150x + 5625)$

75. $(x^2 + 151x + 730)(x^2 + 152x + 5776)$

76. $(x^2 + 153x + 740)(x^2 + 154x + 5929)$

77. $(x^2 + 155x + 750)(x^2 + 156x + 6084)$

78. $(x^2 + 157x + 760)(x^2 + 158x + 6241)$

79. $(x^2 + 159x + 770)(x^2 + 160x + 6400)$

80. $(x^2 + 161x + 780)(x^2 + 162x + 6561)$

81. $(x^2 + 163x + 790)(x^2 + 164x + 6724)$

82. $(x^2 + 165x + 800)(x^2 + 166x + 6889)$

83. $(x^2 + 167x + 810)(x^2 + 168x + 7056)$

84. $(x^2 + 169x + 820)(x^2 + 170x + 7225)$

85. $(x^2 + 171x + 830)(x^2 + 172x + 7396)$

86. $(x^2 + 173x + 840)(x^2 + 174x + 7569)$

87. $(x^2 + 175x + 850)(x^2 + 176x + 7744)$

88. $(x^2 + 177x + 860)(x^2 + 178x + 7921)$

89. $(x^2 + 179x + 870)(x^2 + 180x + 8100)$

90. $(x^2 + 181x + 880)(x^2 + 182x + 8281)$

91. $(x^2 + 183x + 890)(x^2 + 184x + 8464)$

92. $(x^2 + 185x + 900)(x^2 + 186x + 8649)$

93. $(x^2 + 187x + 910)(x^2 + 188x + 8836)$

94. $(x^2 + 189x + 920)(x^2 + 190x + 9025)$

95. $(x^2 + 191x + 930)(x^2 + 192x + 9216)$

96. $(x^2 + 193x + 940)(x^2 + 194x + 9409)$

97. $(x^2 + 195x + 950)(x^2 + 196x + 9604)$

98. $(x^2 + 197x + 960)(x^2 + 198x + 9801)$

99. $(x^2 + 199x + 970)(x^2 + 200x + 10000)$

100. $(x^2 + 201x + 980)(x^2 + 202x + 10201)$

3x2 + 4x2 = 7x2

3x + 4x = 7x

5x2 + 3x2 = 8x2

2x2 + 5x2 + 3x2 = 10x2

4x2 + 3x2 + 2x2 = 9x2

5x2 + 3x2 + 2x2 + 4x2 = 14x2

3x2 + 4x2 + 5x2 + 2x2 = 14x2

2x2 + 3x2 = 5x2

4x2 + 3x2 + 2x2 + 5x2 = 14x2

3x2 + 4x2 + 5x2 + 2x2 = 14x2

5x2 + 3x2 + 4x2 + 2x2 = 14x2

2x2 + 3x2 + 4x2 + 5x2 = 14x2

4x2 + 3x2 + 5x2 + 2x2 = 14x2

3x2 + 4x2 + 5x2 + 2x2 = 14x2

5x2 + 3x2 + 4x2 + 2x2 = 14x2

2x2 + 3x2 + 4x2 + 5x2 = 14x2

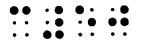
4x2 + 3x2 + 5x2 + 2x2 = 14x2

3x2 + 4x2 + 5x2 + 2x2 = 14x2

5x2 + 3x2 + 4x2 + 2x2 = 14x2

2x2 + 3x2 + 4x2 + 5x2 = 14x2

Braille text consisting of multiple lines of mathematical expressions and symbols, including numbers, plus signs, multiplication signs, and fractions.



Braille text consisting of multiple lines of mathematical expressions and symbols, including numbers and algebraic terms.

1. $(x^2 + 3x + 2)(x^2 + 5x + 6)$

2. $(x^2 + 4x + 4)(x^2 + 6x + 9)$

3. $(x^2 + 7x + 12)(x^2 + 8x + 15)$

4. $(x^2 + 9x + 14)(x^2 + 10x + 21)$

5. $(x^2 + 11x + 30)(x^2 + 12x + 36)$

6. $(x^2 + 13x + 42)(x^2 + 14x + 49)$

7. $(x^2 + 15x + 50)(x^2 + 16x + 64)$

8. $(x^2 + 17x + 60)(x^2 + 18x + 81)$

9. $(x^2 + 19x + 70)(x^2 + 20x + 100)$

10. $(x^2 + 21x + 80)(x^2 + 22x + 121)$

11. $(x^2 + 23x + 90)(x^2 + 24x + 144)$

12. $(x^2 + 25x + 100)(x^2 + 26x + 169)$

13. $(x^2 + 27x + 110)(x^2 + 28x + 196)$

14. $(x^2 + 29x + 120)(x^2 + 30x + 225)$

15. $(x^2 + 31x + 130)(x^2 + 32x + 256)$

16. $(x^2 + 33x + 140)(x^2 + 34x + 289)$

17. $(x^2 + 35x + 150)(x^2 + 36x + 324)$

18. $(x^2 + 37x + 160)(x^2 + 38x + 361)$

19. $(x^2 + 39x + 170)(x^2 + 40x + 400)$

20. $(x^2 + 41x + 180)(x^2 + 42x + 441)$

21. $(x^2 + 43x + 190)(x^2 + 44x + 484)$

22. $(x^2 + 45x + 200)(x^2 + 46x + 529)$

23. $(x^2 + 47x + 210)(x^2 + 48x + 576)$

24. $(x^2 + 49x + 220)(x^2 + 50x + 625)$

25. $(x^2 + 51x + 230)(x^2 + 52x + 676)$

26. $(x^2 + 53x + 240)(x^2 + 54x + 729)$

27. $(x^2 + 55x + 250)(x^2 + 56x + 784)$

28. $(x^2 + 57x + 260)(x^2 + 58x + 841)$

29. $(x^2 + 59x + 270)(x^2 + 60x + 900)$

30. $(x^2 + 61x + 280)(x^2 + 62x + 961)$

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36. $(x^2 + 73x + 340)(x^2 + 74x + 1369)$

37. $(x^2 + 75x + 350)(x^2 + 76x + 1444)$

38. $(x^2 + 77x + 360)(x^2 + 78x + 1521)$

39. $(x^2 + 79x + 370)(x^2 + 80x + 1600)$

40. $(x^2 + 81x + 380)(x^2 + 82x + 1681)$

41. $(x^2 + 83x + 390)(x^2 + 84x + 1764)$

42. $(x^2 + 85x + 400)(x^2 + 86x + 1849)$

43. $(x^2 + 87x + 410)(x^2 + 88x + 1936)$

44. $(x^2 + 89x + 420)(x^2 + 90x + 2025)$

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49. $(x^2 + 99x + 470)(x^2 + 100x + 2500)$

50. $(x^2 + 101x + 480)(x^2 + 102x + 2601)$

$(x^2 + 2x + 1)(x^2 + 3x + 2)$
 $(x^2 + 2x + 1)(x + 1)(x + 2)$
 $(x + 1)^2(x + 2)$
 $(x + 1)(x + 2)(x + 1)$
 $(x + 1)(x + 2)(x + 1)$
 $(x + 1)(x + 2)(x + 1)$

$(x^2 + 2x + 1)(x^2 + 3x + 2)$
 $(x^2 + 2x + 1)(x + 1)(x + 2)$
 $(x + 1)^2(x + 2)$

$(x^2 + 2x + 1)(x^2 + 3x + 2)$
 $(x^2 + 2x + 1)(x + 1)(x + 2)$
 $(x + 1)^2(x + 2)$
 $(x + 1)(x + 2)(x + 1)$



Braille text consisting of multiple lines of mathematical expressions and symbols, including numbers, plus signs, multiplication signs, and fractions.

$(2x + 3)^2 - (x - 4)^2$

$= (2x + 3)(2x + 3) - (x - 4)(x - 4)$

$= 4x^2 + 12x + 9 - (x^2 - 8x + 16)$

$= 4x^2 + 12x + 9 - x^2 + 8x - 16$

$= 3x^2 + 20x - 7$

$(x + 2)^2 - (x - 3)^2$

$= (x + 2)(x + 2) - (x - 3)(x - 3)$

$= x^2 + 4x + 4 - (x^2 - 6x + 9)$

$= x^2 + 4x + 4 - x^2 + 6x - 9$

$= 10x - 5$

$(x + 5)^2 - (x - 2)^2$

$= (x + 5)(x + 5) - (x - 2)(x - 2)$

$= x^2 + 10x + 25 - (x^2 - 4x + 4)$

$= x^2 + 10x + 25 - x^2 + 4x - 4$

$= 14x + 21$

$(x + 3)^2 - (x - 1)^2$

$= (x + 3)(x + 3) - (x - 1)(x - 1)$

$= x^2 + 6x + 9 - (x^2 - 2x + 1)$

$= x^2 + 6x + 9 - x^2 + 2x - 1$

$= 8x + 8$

$(x + 4)^2 - (x - 3)^2$

$= (x + 4)(x + 4) - (x - 3)(x - 3)$

$= x^2 + 8x + 16 - (x^2 - 6x + 9)$

$= x^2 + 8x + 16 - x^2 + 6x - 9$

$= 14x + 7$

$(x + 6)^2 - (x - 4)^2$

$= (x + 6)(x + 6) - (x - 4)(x - 4)$

$= x^2 + 12x + 36 - (x^2 - 8x + 16)$

$= x^2 + 12x + 36 - x^2 + 8x - 16$

$= 20x + 20$

$3x^2 + 2x - 5$ $4x^2 - 7x + 1$ $5x^2 + 3x - 2$ $6x^2 - 4x + 3$ $7x^2 + 5x - 1$ $8x^2 - 2x + 4$ $9x^2 + 1x - 3$ $10x^2 - 6x + 2$

$11x^2 + 4x - 5$ $12x^2 - 3x + 2$ $13x^2 + 2x - 1$ $14x^2 - 1x + 3$ $15x^2 + 0x - 4$ $16x^2 - 5x + 1$ $17x^2 + 6x - 2$ $18x^2 - 7x + 3$

$19x^2 + 8x - 3$ $20x^2 - 9x + 4$ $21x^2 + 10x - 5$ $22x^2 - 11x + 6$ $23x^2 + 12x - 7$ $24x^2 - 13x + 8$ $25x^2 + 14x - 9$ $26x^2 - 15x + 10$

$27x^2 + 16x - 11$ $28x^2 - 17x + 12$ $29x^2 + 18x - 13$ $30x^2 - 19x + 14$ $31x^2 + 20x - 15$ $32x^2 - 21x + 16$ $33x^2 + 22x - 17$ $34x^2 - 23x + 18$

$35x^2 + 24x - 19$ $36x^2 - 25x + 20$ $37x^2 + 26x - 21$ $38x^2 - 27x + 22$ $39x^2 + 28x - 23$ $40x^2 - 29x + 24$ $41x^2 + 30x - 25$ $42x^2 - 31x + 26$

$43x^2 + 32x - 27$ $44x^2 - 33x + 28$ $45x^2 + 34x - 29$ $46x^2 - 35x + 30$ $47x^2 + 36x - 31$ $48x^2 - 37x + 32$ $49x^2 + 38x - 33$ $50x^2 - 39x + 34$

$51x^2 + 40x - 35$

1. $(x^2 + 3x + 2)(x^2 + 4x + 4)$

2. $(x^2 + 5x + 6)(x^2 + 7x + 12)$

3. $(x^2 + 2x + 1)(x^2 + 3x + 2)$

4. $(x^2 + 4x + 4)(x^2 + 5x + 6)$

5. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)$

6. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)$

7. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)$

8. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)$

9. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)$

10. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)$

11. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)$

12. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)$

13. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)$

14. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)$

15. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)$

16. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)$

17. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)$

18. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)$

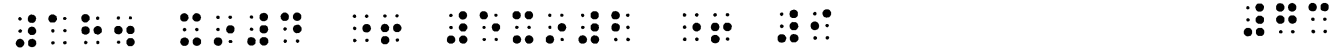
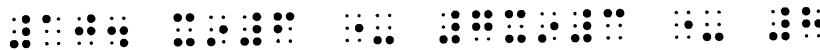
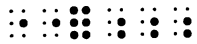
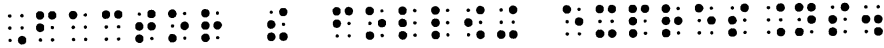
19. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)(x^2 + 11x + 30)$

20. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)(x^2 + 11x + 30)$

21. $(x^2 + 2x + 1)(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)(x^2 + 11x + 30)$

22. $(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)(x^2 + 11x + 30)(x^2 + 12x + 36)$

23. $(x^2 + 3x + 2)(x^2 + 4x + 4)(x^2 + 5x + 6)(x^2 + 7x + 12)(x^2 + 8x + 16)(x^2 + 9x + 20)(x^2 + 10x + 25)(x^2 + 11x + 30)(x^2 + 12x + 36)$



1. $(x^2 + 3x + 2)(x^2 + 4x + 4)$

$(x^2 + 3x + 2)(x^2 + 4x + 4)$

$x^2(x^2 + 4x + 4) + 3x(x^2 + 4x + 4) + 2(x^2 + 4x + 4)$

$x^4 + 4x^3 + 4x^2 + 3x^3 + 12x^2 + 12x + 2x^2 + 8x + 8$

$x^4 + 7x^3 + 16x^2 + 20x + 8$

$(x^2 + 3x + 2)(x^2 + 4x + 4)$

$(x^2 + 3x + 2)(x^2 + 4x + 4)$

$x^2(x^2 + 4x + 4) + 3x(x^2 + 4x + 4) + 2(x^2 + 4x + 4)$

$x^4 + 4x^3 + 4x^2 + 3x^3 + 12x^2 + 12x + 2x^2 + 8x + 8$

$x^4 + 7x^3 + 16x^2 + 20x + 8$

1. $2x^2 + 3x - 5$ and $x^2 - 4x + 7$ are two polynomials.
a. Find the sum of these two polynomials.
b. Find the difference of these two polynomials.
c. Find the product of these two polynomials.
d. Find the quotient of these two polynomials.
e. Find the remainder when $2x^2 + 3x - 5$ is divided by $x^2 - 4x + 7$.

2. A rectangular garden has a length of $2x + 3$ and a width of $x + 5$.
a. Find the area of the garden.
b. Find the perimeter of the garden.
c. Find the length of the garden if the width is 10 units.
d. Find the width of the garden if the length is 15 units.

3. A rectangular garden has a length of $2x + 3$ and a width of $x + 5$.
a. Find the area of the garden.
b. Find the perimeter of the garden.
c. Find the length of the garden if the width is 10 units.
d. Find the width of the garden if the length is 15 units.

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1. $(x^2 + 3x + 2)(x^2 + 5x + 6)$

$(x^2 + 3x + 2)(x^2 + 5x + 6)$

$x^2(x^2 + 5x + 6) + 3x(x^2 + 5x + 6) + 2(x^2 + 5x + 6)$

$x^4 + 5x^3 + 6x^2 + 3x^3 + 15x^2 + 18x + 2x^2 + 10x + 12$

$x^4 + 8x^3 + 14x^2 + 28x + 12$

2. $(x^2 + 4x + 4)(x^2 + 6x + 9)$

$(x^2 + 4x + 4)(x^2 + 6x + 9)$

$x^2(x^2 + 6x + 9) + 4x(x^2 + 6x + 9) + 4(x^2 + 6x + 9)$

$x^4 + 6x^3 + 9x^2 + 4x^3 + 24x^2 + 36x + 4x^2 + 24x + 36$

$x^4 + 10x^3 + 17x^2 + 60x + 36$

3. $(x^2 + 7x + 12)(x^2 + 8x + 15)$

$(x^2 + 7x + 12)(x^2 + 8x + 15)$

$x^2(x^2 + 8x + 15) + 7x(x^2 + 8x + 15) + 12(x^2 + 8x + 15)$

$x^4 + 8x^3 + 15x^2 + 7x^3 + 56x^2 + 105x + 12x^2 + 96x + 180$

$x^4 + 15x^3 + 27x^2 + 201x + 180$

4. $(x^2 + 9x + 14)(x^2 + 10x + 21)$

$(x^2 + 9x + 14)(x^2 + 10x + 21)$

$x^2(x^2 + 10x + 21) + 9x(x^2 + 10x + 21) + 14(x^2 + 10x + 21)$

$x^4 + 10x^3 + 21x^2 + 9x^3 + 90x^2 + 189x + 14x^2 + 140x + 294$

$x^4 + 19x^3 + 35x^2 + 329x + 294$

5. $(x^2 + 11x + 18)(x^2 + 12x + 20)$

$(x^2 + 11x + 18)(x^2 + 12x + 20)$

$x^2(x^2 + 12x + 20) + 11x(x^2 + 12x + 20) + 18(x^2 + 12x + 20)$

$x^4 + 12x^3 + 20x^2 + 11x^3 + 132x^2 + 220x + 18x^2 + 216x + 360$

$x^4 + 23x^3 + 48x^2 + 436x + 360$

6. $(x^2 + 13x + 20)(x^2 + 14x + 24)$

$(x^2 + 13x + 20)(x^2 + 14x + 24)$

$x^2(x^2 + 14x + 24) + 13x(x^2 + 14x + 24) + 20(x^2 + 14x + 24)$

$x^4 + 14x^3 + 24x^2 + 13x^3 + 182x^2 + 312x + 20x^2 + 280x + 480$

$x^4 + 27x^3 + 54x^2 + 592x + 480$

7. $(x^2 + 15x + 24)(x^2 + 16x + 28)$

$(x^2 + 15x + 24)(x^2 + 16x + 28)$

$x^2(x^2 + 16x + 28) + 15x(x^2 + 16x + 28) + 24(x^2 + 16x + 28)$

$x^4 + 16x^3 + 28x^2 + 15x^3 + 240x^2 + 420x + 24x^2 + 384x + 672$

$x^4 + 31x^3 + 62x^2 + 804x + 672$

8. $(x^2 + 17x + 30)(x^2 + 18x + 32)$

$(x^2 + 17x + 30)(x^2 + 18x + 32)$

$x^2(x^2 + 18x + 32) + 17x(x^2 + 18x + 32) + 30(x^2 + 18x + 32)$

$x^4 + 18x^3 + 32x^2 + 17x^3 + 306x^2 + 540x + 30x^2 + 540x + 960$

$x^4 + 35x^3 + 70x^2 + 1080x + 960$

9. $(x^2 + 19x + 36)(x^2 + 20x + 36)$

$(x^2 + 19x + 36)(x^2 + 20x + 36)$

$x^2(x^2 + 20x + 36) + 19x(x^2 + 20x + 36) + 36(x^2 + 20x + 36)$

$x^4 + 20x^3 + 36x^2 + 19x^3 + 380x^2 + 684x + 36x^2 + 720x + 1296$

$x^4 + 39x^3 + 78x^2 + 1404x + 1296$

10. $(x^2 + 21x + 42)(x^2 + 22x + 42)$

$(x^2 + 21x + 42)(x^2 + 22x + 42)$

$x^2(x^2 + 22x + 42) + 21x(x^2 + 22x + 42) + 42(x^2 + 22x + 42)$

$x^4 + 22x^3 + 42x^2 + 21x^3 + 462x^2 + 882x + 42x^2 + 882x + 1764$

$x^4 + 43x^3 + 84x^2 + 1764x + 1764$

$(x^2 + 3x + 2)(x + 4)$

$(x^2 + 3x + 2)(x + 4)$

$x^3 + 4x^2 + 3x^2 + 12x + 2x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

$x^3 + 7x^2 + 14x + 8$

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$(x^2 + 3x + 2)(x + 4)$

$(x^2 + 3x + 2)(x + 4)$

1. $x^2 + 3x + 2 = (x+1)(x+2)$

2. $x^2 + 5x + 6 = (x+2)(x+3)$

3. $x^2 + 7x + 12 = (x+3)(x+4)$

4. $x^2 + 9x + 18 = (x+3)(x+6)$

5. $x^2 + 11x + 28 = (x+4)(x+7)$

6. $x^2 + 13x + 40 = (x+5)(x+8)$

7. $x^2 + 15x + 50 = (x+5)(x+10)$

8. $x^2 + 17x + 60 = (x+5)(x+12)$

9. $x^2 + 19x + 70 = (x+7)(x+14)$

10. $x^2 + 21x + 80 = (x+8)(x+16)$

11. $x^2 + 23x + 90 = (x+9)(x+18)$

12. $x^2 + 25x + 100 = (x+10)(x+20)$

13. $x^2 + 27x + 112 = (x+7)(x+28)$

14. $x^2 + 29x + 120 = (x+8)(x+30)$

15. $x^2 + 31x + 130 = (x+10)(x+31)$

16. $x^2 + 33x + 140 = (x+10)(x+33)$

17. $x^2 + 35x + 150 = (x+10)(x+35)$

18. $x^2 + 37x + 160 = (x+10)(x+37)$

19. $x^2 + 39x + 170 = (x+10)(x+39)$

20. $x^2 + 41x + 180 = (x+10)(x+41)$

21. $x^2 + 43x + 190 = (x+10)(x+43)$

22. $x^2 + 45x + 200 = (x+10)(x+45)$

23. $x^2 + 47x + 210 = (x+10)(x+47)$

24. $x^2 + 49x + 220 = (x+10)(x+49)$

25. $x^2 + 51x + 230 = (x+10)(x+51)$

26. $x^2 + 53x + 240 = (x+10)(x+53)$

27. $x^2 + 55x + 250 = (x+10)(x+55)$

28. $x^2 + 57x + 260 = (x+10)(x+57)$

29. $x^2 + 59x + 270 = (x+10)(x+59)$

30. $x^2 + 61x + 280 = (x+10)(x+61)$

31. $x^2 + 63x + 290 = (x+10)(x+63)$

32. $x^2 + 65x + 300 = (x+10)(x+65)$

33. $x^2 + 67x + 310 = (x+10)(x+67)$

34. $x^2 + 69x + 320 = (x+10)(x+69)$

35. $x^2 + 71x + 330 = (x+10)(x+71)$

36. $x^2 + 73x + 340 = (x+10)(x+73)$

37. $x^2 + 75x + 350 = (x+10)(x+75)$

38. $x^2 + 77x + 360 = (x+10)(x+77)$

39. $x^2 + 79x + 370 = (x+10)(x+79)$

40. $x^2 + 81x + 380 = (x+10)(x+81)$

41. $x^2 + 83x + 390 = (x+10)(x+83)$

42. $x^2 + 85x + 400 = (x+10)(x+85)$

43. $x^2 + 87x + 410 = (x+10)(x+87)$

44. $x^2 + 89x + 420 = (x+10)(x+89)$

45. $x^2 + 91x + 430 = (x+10)(x+91)$

46. $x^2 + 93x + 440 = (x+10)(x+93)$

47. $x^2 + 95x + 450 = (x+10)(x+95)$

48. $x^2 + 97x + 460 = (x+10)(x+97)$

49. $x^2 + 99x + 470 = (x+10)(x+99)$

50. $x^2 + 101x + 480 = (x+10)(x+101)$

51. $x^2 + 103x + 490 = (x+10)(x+103)$

52. $x^2 + 105x + 500 = (x+10)(x+105)$

53. $x^2 + 107x + 510 = (x+10)(x+107)$

54. $x^2 + 109x + 520 = (x+10)(x+109)$

55. $x^2 + 111x + 530 = (x+10)(x+111)$

56. $x^2 + 113x + 540 = (x+10)(x+113)$

57. $x^2 + 115x + 550 = (x+10)(x+115)$

58. $x^2 + 117x + 560 = (x+10)(x+117)$

59. $x^2 + 119x + 570 = (x+10)(x+119)$

60. $x^2 + 121x + 580 = (x+10)(x+121)$

61. $x^2 + 123x + 590 = (x+10)(x+123)$

62. $x^2 + 125x + 600 = (x+10)(x+125)$

63. $x^2 + 127x + 610 = (x+10)(x+127)$

64. $x^2 + 129x + 620 = (x+10)(x+129)$

65. $x^2 + 131x + 630 = (x+10)(x+131)$

66. $x^2 + 133x + 640 = (x+10)(x+133)$

67. $x^2 + 135x + 650 = (x+10)(x+135)$

68. $x^2 + 137x + 660 = (x+10)(x+137)$

69. $x^2 + 139x + 670 = (x+10)(x+139)$

70. $x^2 + 141x + 680 = (x+10)(x+141)$

71. $x^2 + 143x + 690 = (x+10)(x+143)$

72. $x^2 + 145x + 700 = (x+10)(x+145)$

73. $x^2 + 147x + 710 = (x+10)(x+147)$

74. $x^2 + 149x + 720 = (x+10)(x+149)$

75. $x^2 + 151x + 730 = (x+10)(x+151)$

76. $x^2 + 153x + 740 = (x+10)(x+153)$

77. $x^2 + 155x + 750 = (x+10)(x+155)$

78. $x^2 + 157x + 760 = (x+10)(x+157)$

79. $x^2 + 159x + 770 = (x+10)(x+159)$

80. $x^2 + 161x + 780 = (x+10)(x+161)$

81. $x^2 + 163x + 790 = (x+10)(x+163)$

82. $x^2 + 165x + 800 = (x+10)(x+165)$

83. $x^2 + 167x + 810 = (x+10)(x+167)$

84. $x^2 + 169x + 820 = (x+10)(x+169)$

85. $x^2 + 171x + 830 = (x+10)(x+171)$

86. $x^2 + 173x + 840 = (x+10)(x+173)$

87. $x^2 + 175x + 850 = (x+10)(x+175)$

88. $x^2 + 177x + 860 = (x+10)(x+177)$

89. $x^2 + 179x + 870 = (x+10)(x+179)$

90. $x^2 + 181x + 880 = (x+10)(x+181)$

91. $x^2 + 183x + 890 = (x+10)(x+183)$

92. $x^2 + 185x + 900 = (x+10)(x+185)$

93. $x^2 + 187x + 910 = (x+10)(x+187)$

94. $x^2 + 189x + 920 = (x+10)(x+189)$

95. $x^2 + 191x + 930 = (x+10)(x+191)$

96. $x^2 + 193x + 940 = (x+10)(x+193)$

97. $x^2 + 195x + 950 = (x+10)(x+195)$

98. $x^2 + 197x + 960 = (x+10)(x+197)$

99. $x^2 + 199x + 970 = (x+10)(x+199)$

100. $x^2 + 201x + 980 = (x+10)(x+201)$

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$(x^2 + 3x + 2)(x + 4) = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$(x^2 + 3x + 2)(x + 4) = x^3 + 4x^2 + 3x^2 + 12x + 2x + 8 = x^3 + 7x^2 + 14x + 8$

$$\frac{3x^2 - 4x + 5}{x^2 - 2x + 1} \div \frac{x^2 - 3x + 2}{x^2 - 4x + 4} \cdot \frac{x^2 + 2x + 1}{x^2 - 1}$$

$$= \frac{3x^2 - 4x + 5}{(x-1)^2} \cdot \frac{(x-2)(x-1)}{(x-2)^2} \cdot \frac{(x+1)(x+1)}{(x-1)(x+1)}$$

$$= \frac{3x^2 - 4x + 5}{(x-1)^2} \cdot \frac{(x-1)}{(x-2)} \cdot \frac{(x+1)}{1}$$

$$= \frac{(3x^2 - 4x + 5)(x-1)(x+1)}{(x-1)^2(x-2)}$$

$$= \frac{(3x^2 - 4x + 5)(x+1)}{(x-1)(x-2)}$$

$$\frac{(3x^2 - 4x + 5)(x+1)}{(x-1)(x-2)}$$

Braille text consisting of multiple lines of characters, including mathematical notation and symbols such as plus, minus, multiplication, and division signs, and numbers.

Braille equation line 1

Braille equation line 2

Braille equation line 3

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Braille equation line 8

Braille equation line 9

Braille equation line 10

Braille equation line 11

Braille equation line 12

Braille equation line 13

Braille equation line 14

Braille equation line 15

Braille equation line 16

Braille equation line 17

Braille equation line 18

Braille equation line 19

$(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

$(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

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$(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

$(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

$(x + 2)(x + 3)(x + 4)(x + 5)(x + 6)$

1. $x^2 + 4x + 4 = (x + 2)^2$
 $x^2 + 4x + 4 = (x + 2)(x + 2)$
 $x^2 + 4x + 4 = (x + 2)^2$

2. $x^2 + 6x + 9 = (x + 3)^2$
 $x^2 + 6x + 9 = (x + 3)(x + 3)$
 $x^2 + 6x + 9 = (x + 3)^2$

3. $x^2 + 8x + 16 = (x + 4)^2$
 $x^2 + 8x + 16 = (x + 4)(x + 4)$
 $x^2 + 8x + 16 = (x + 4)^2$

4. $x^2 + 10x + 25 = (x + 5)^2$
 $x^2 + 10x + 25 = (x + 5)(x + 5)$
 $x^2 + 10x + 25 = (x + 5)^2$

5. $x^2 + 12x + 36 = (x + 6)^2$
 $x^2 + 12x + 36 = (x + 6)(x + 6)$
 $x^2 + 12x + 36 = (x + 6)^2$

6. $x^2 + 14x + 49 = (x + 7)^2$
 $x^2 + 14x + 49 = (x + 7)(x + 7)$
 $x^2 + 14x + 49 = (x + 7)^2$

7. $x^2 + 16x + 64 = (x + 8)^2$
 $x^2 + 16x + 64 = (x + 8)(x + 8)$
 $x^2 + 16x + 64 = (x + 8)^2$

8. $x^2 + 18x + 81 = (x + 9)^2$
 $x^2 + 18x + 81 = (x + 9)(x + 9)$
 $x^2 + 18x + 81 = (x + 9)^2$

9. $x^2 + 20x + 100 = (x + 10)^2$
 $x^2 + 20x + 100 = (x + 10)(x + 10)$
 $x^2 + 20x + 100 = (x + 10)^2$

10. $x^2 + 22x + 121 = (x + 11)^2$
 $x^2 + 22x + 121 = (x + 11)(x + 11)$
 $x^2 + 22x + 121 = (x + 11)^2$



1. $(x^2 + 3x - 4)^2$

2. $(x^2 + 2x - 1)^2$

3. $(x^2 + 4x + 4)^2$

4. $(x^2 + 5x + 6)^2$

5. $(x^2 + 6x + 9)^2$

6. $(x^2 + 7x + 10)^2$

7. $(x^2 + 8x + 12)^2$

8. $(x^2 + 9x + 14)^2$

9. $(x^2 + 10x + 16)^2$

10. $(x^2 + 11x + 18)^2$

11. $(x^2 + 12x + 20)^2$

12. $(x^2 + 13x + 22)^2$

13. $(x^2 + 14x + 24)^2$

14. $(x^2 + 15x + 26)^2$

15. $(x^2 + 16x + 28)^2$

16. $(x^2 + 17x + 30)^2$

17. $(x^2 + 18x + 32)^2$

18. $(x^2 + 19x + 34)^2$

19. $(x^2 + 20x + 36)^2$

20. $(x^2 + 21x + 38)^2$

21. $(x^2 + 22x + 40)^2$

22. $(x^2 + 23x + 42)^2$

23. $(x^2 + 24x + 44)^2$

$$(x+2)(x-3) = x^2 - 3x + 2x - 6 = x^2 - x - 6$$

$$(x+5)(x-4) = x^2 - 4x + 5x - 20 = x^2 + x - 20$$

$$(x+1)(x-2) = x^2 - 2x + x - 2 = x^2 - x - 2$$

$$(x+3)(x-1) = x^2 - x + 3x - 3 = x^2 + 2x - 3$$

$$(x+4)(x-3) = x^2 - 3x + 4x - 12 = x^2 + x - 12$$

$$(x+6)(x-2) = x^2 - 2x + 6x - 12 = x^2 + 4x - 12$$

$$(x+2)(x-5) = x^2 - 5x + 2x - 10 = x^2 - 3x - 10$$

$$(x+1)(x-4) = x^2 - 4x + x - 4 = x^2 - 3x - 4$$

$$(x+3)(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$$

$$(x+4)(x-1) = x^2 - x + 4x - 4 = x^2 + 3x - 4$$

$$(x+5)(x-3) = x^2 - 3x + 5x - 15 = x^2 + 2x - 15$$

$$(x+2)(x-6) = x^2 - 6x + 2x - 12 = x^2 - 4x - 12$$

$$(x+3)(x-4) = x^2 - 4x + 3x - 12 = x^2 - x - 12$$

$$(x+4)(x-2) = x^2 - 2x + 4x - 8 = x^2 + 2x - 8$$

$$(x+1)(x-5) = x^2 - 5x + x - 5 = x^2 - 4x - 5$$

$$(x+2)(x-1)$$

$$(x+3)(x-2) = x^2 - 2x + 3x - 6 = x^2 + x - 6$$



Sample 3. Calculus

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left-hand pages) and the Unified English Braille Code as of June 2001 (on right-hand pages).

5

Vector Integral Calculus

Part I. Two-Dimensional Theory

5.1 INTRODUCTION

The topic of this chapter is *line and surface integrals*. It will be seen that these can both be regarded as integrals of vectors and that the principal theorems can be most simply stated in terms of vectors; hence the title "vector integral calculus."

A familiar line integral is that of arc length: $\int_C ds$. The subscript C indicates that one is measuring the length of a curve C , as in Fig. 5.1. If C is given in parametric form $x = x(t)$, $y = y(t)$, the line integral reduces to the ordinary definite integral:

$$\int_C ds = \int_{t_1}^{t_2} \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt.$$

If the curve C represents a wire whose density (mass per unit length) varies along C , then the wire has a total mass

$$M = \int_C f(x, y) ds,$$

where $f(x, y)$ is the density at the point (x, y) of the wire. The new integral can be expressed in terms of a parameter as previously or can be thought of simply as a limit of a sum

$$\int_C f(x, y) ds = \lim_{n \rightarrow \infty} \sum_{i=1}^n f(x_i^*, y_i^*) \Delta_i s.$$

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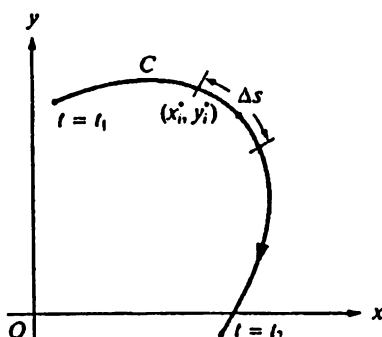


Figure 5.1 Line integral.

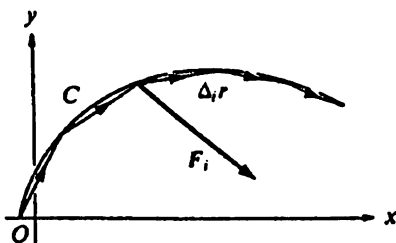
Here the curve has been subdivided into n pieces of lengths $\Delta_1 s, \Delta_2 s, \dots, \Delta_n s$, and the point (x_i^*, y_i^*) lies on the i th piece. The limit is taken as n becomes infinite, while the maximum $\Delta_i s$ approaches 0.

A third example of a line integral is that of *work*. If a particle moves from one end of C to the other under the influence of a force F , the work done by this force is defined as

$$\int_C F_T ds,$$

where F_T denotes the component of F on the tangent T in the direction of motion. This integral can be thought of as a limit of a sum as previously. However, another interpretation is possible. We first remark that the work done by a constant force F in moving a particle from A to B on the line segment AB is $F \cdot \overline{AB}$; for this scalar product is equal to $|F| \cdot \cos \alpha \cdot |\overline{AB}|$, α being the angle between F and \overline{AB} , and hence to the product of force component in direction of motion by the distance moved. Now the motion of the particle along C can be thought of as the sum of many small displacements along line segments, as suggested in Fig. 5.2. If these displacements are denoted by $\Delta_1 r, \Delta_2 r, \dots, \Delta_n r$, the work done would be approximated by a sum of form

$$\sum_{i=1}^n F_i \cdot \Delta_i r,$$

Figure 5.2 Work = $\int_C F \cdot dr$.

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where F_i is the force acting for the i th displacement. The limiting form of this is again equal to the line integral $\int F_T ds$, but because of the way the limit is obtained, we can also write it as

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

One can thus write

$$\text{work} = \int_C F_T ds = \int_C \mathbf{F} \cdot d\mathbf{r}.$$

If the displacement vector $\Delta\mathbf{r}$ and force \mathbf{F} are expressed in components,

$$\mathbf{F} = F_x \mathbf{i} + F_y \mathbf{j}, \quad \Delta\mathbf{r} = \Delta x \mathbf{i} + \Delta y \mathbf{j},$$

the element of work $\mathbf{F} \cdot \Delta\mathbf{r}$ becomes

$$\mathbf{F} \cdot \Delta\mathbf{r} = F_x \Delta x + F_y \Delta y.$$

The total amount of work done is then approximated by a sum of form

$$\sum (F_x \Delta x + F_y \Delta y) = \sum F_x \Delta x + \sum F_y \Delta y.$$

The limiting form of this is a sum of two integrals:

$$\int_C F_x dx + \int_C F_y dy.$$

The first integral represents the work done by the x -component of the force; the second integral represents the work done by the y -component of the force.

It thus appears that one has three types of line integrals to consider, namely, the types

$$\int_C f(x, y) ds, \quad \int_C P(x, y) dx, \quad \int_C Q(x, y) dy,$$

which are limits of sums

$$\sum f(x, y) \Delta s, \quad \sum P(x, y) \Delta x, \quad \sum Q(x, y) \Delta y.$$

The foregoing gives the basis for the theory of line integrals in the plane. A very slight extension of these ideas leads to line integrals in space:

$$\int_C f(x, y, z) ds, \quad \int_C f(x, y, z) dx, \quad \dots$$

Surface integrals appear as a natural generalization, with the surface area element $d\sigma$ replacing the arc element ds :

$$\int_S f(x, y, z) d\sigma = \lim \sum f(x, y, z) \Delta\sigma.$$

There are corresponding component integrals

$$\int_S \int f(x, y, z) dx dy, \quad \int_S \int f(x, y, z) dy dz, \quad \dots$$

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and a vector surface integral

$$\iint_S \mathbf{F} \cdot d\boldsymbol{\sigma} = \iint_S (\mathbf{F} \cdot \mathbf{n}) d\sigma,$$

where $d\boldsymbol{\sigma} = \mathbf{n} d\sigma$ is the "area element vector," \mathbf{n} being a unit normal vector to the surface.

It will be seen that the basic theorems—those of Green, Gauss, and Stokes—concern the relations between line, surface, and volume (triple) integrals. These correspond to fundamental physical relations between such quantities as flux, circulation, divergence, and curl. The applications will be considered at the end of the chapter.

5.2 LINE INTEGRALS IN THE PLANE

We now state in precise form the definitions outlined in the preceding section.

By a *smooth curve* C in the xy -plane will be meant a curve representable in the form:

$$x = \phi(t), \quad y = \psi(t), \quad h \leq t \leq k, \quad (5.1)$$

where x and y are continuous and have continuous derivatives for $h \leq t \leq k$. The curve C can be assigned a direction, which will usually be that of increasing t . If A denotes the point $[\phi(h), \psi(h)]$ and B denotes the point $[\phi(k), \psi(k)]$, then C can be thought of as the path of a point moving continuously from A to B . This path may cross itself, as for the curve C_1 of Fig. 5.3. If the initial point A and terminal point B coincide, C is termed a *closed curve*; if, in addition, (x, y) moves from A to $B = A$ without retracing any other point, C is called a *simple closed curve* (curve C_2 of Fig. 5.3).

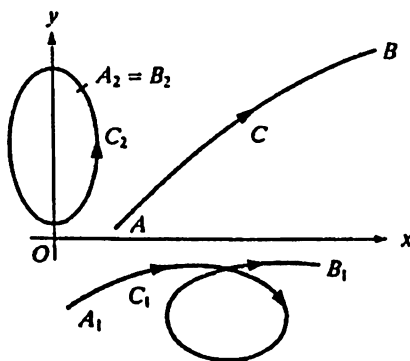


Figure 5.3 Paths of integration.

Let C be a smooth curve as previously, with positive direction that of increasing t . Let $f(x, y)$ be a function defined at least when (x, y) is on C . The

line integral $\int_C f(x, y) dx$ is defined as a limit:

$$\int_C f(x, y) dx = \lim \sum_{i=1}^n f(x_i^*, y_i^*) \Delta_i x. \quad (5.2)$$

The limit refers to a subdivision of C as indicated in Fig. 5.4. The successive subdivision points are $A: (x_0, y_0), (x_1, y_1), \dots, B: (x_n, y_n)$. These correspond to parameter values: $h = t_0 < t_1 < \dots < t_n = k$. The point (x_i^*, y_i^*) is some point of C between (x_{i-1}, y_{i-1}) and (x_i, y_i) ; that is, (x_i^*, y_i^*) corresponds to a parameter value t_i^* , where $t_{i-1} \leq t_i^* \leq t_i$. $\Delta_i x$ denotes the difference $x_i - x_{i-1}$. The limit is taken as n becomes infinite and the largest $\Delta_i t$ approaches 0, where $\Delta_i t = t_i - t_{i-1}$. Similarly,

$$\int_C f(x, y) dy = \lim \sum f(x_i^*, y_i^*) \Delta_i y, \quad (5.3)$$

where $\Delta_i y = y_i - y_{i-1}$.

The significance of these definitions is guaranteed by the following basic theorems:

I If $f(x, y)$ is continuous on C , then

$$\int_C f(x, y) dx \quad \text{and} \quad \int_C f(x, y) dy \quad \text{exist.}$$

II If $f(x, y)$ is continuous on C , then

$$\int_C f(x, y) dx = \int_h^k f[\phi(t), \psi(t)] \phi'(t) dt, \quad (5.4)$$

$$\int_C f(x, y) dy = \int_h^k f[\phi(t), \psi(t)] \psi'(t) dt. \quad (5.5)$$

Formulas (5.4) and (5.5) reduce the integrals to ordinary definite integrals and are thus essential for computation of particular integrals. Thus let C be the

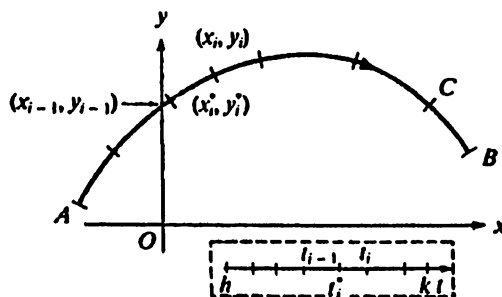


Figure 5.4 Definition of line integral.

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path $x = 1 + t$, $y = t^2$, $0 \leq t \leq 1$, directed with increasing t . Then

$$\int_C (x^2 - y^2) dx = \int_0^1 [(1+t)^2 - t^4] dt = \frac{27}{15},$$

$$\int_C (x^2 - y^2) dy = \int_0^1 [(1+t)^2 - t^4] 2t dt = 2\frac{1}{2}.$$

It is logically easier to prove II first, for I is an immediate consequence of II. To prove II, one notes that the sum $\sum f(x_i^*, y_i^*) \Delta_i x$ can be written as

$$\sum_{i=1}^n f[\phi(t_i^*), \psi(t_i^*)] \frac{\Delta_i x}{\Delta_i t} \Delta_i t.$$

Now $\Delta_i x = x_i - x_{i-1} = \phi'(t_i^{**}) \Delta_i t$ by the Law of the Mean. Hence the sum can be written as

$$\sum_{i=1}^n F(t_i^*) \phi'(t_i^{**}) \Delta_i t,$$

where $F(t) = f[\phi(t), \psi(t)]$ and t_i^* and t_i^{**} are both between t_{i-1} and t_i . It is easily shown [see CLA, Section 12-25] that this sum approaches as limit the integral

$$\int_a^b F(t) \phi'(t) dt = \int_a^b f[\phi(t), \psi(t)] \phi'(t) dt$$

as required. Formula (5.5) is proved in the same way.

We remark that the value of a line integral on C does not depend on the particular parametrization of C , but only on the order in which the points of C are traced. (See Problem 5.)

In many applications the path C is not itself smooth but is composed of a finite number of arcs, each of which is smooth. Thus C might be a broken line. In this case, C is termed *piecewise smooth*. The line integral along C is simply, by definition, the sum of the integrals along the pieces. One verifies at once that (5.2), (5.3), and the theorems I and II continue to hold. In (5.4) and (5.5) the functions $\phi'(t)$ and $\psi'(t)$ will have jump discontinuities, which will not interfere with the existence of the integral (cf. Section 4.1). *Throughout this book all paths of integration for line integrals will be piecewise smooth unless otherwise specified.*

If the curve C is represented in the form

$$y = g(x), \quad a \leq x \leq b,$$

then one can regard x itself as parameter, replacing t ; that is, C is given by the equations

$$x = x, \quad y = g(x), \quad a \leq x \leq b$$

in terms of the parameter x . If the direction of C is that of increasing x , (5.4) and

(5.5) become

$$\int_C f(x, y) dx = \int_a^b f[x, g(x)] dx, \quad (5.6)$$

$$\int_C f(x, y) dy = \int_a^b f[x, g(x)] g'(x) dx. \quad (5.7)$$

The ordinary definite integral $\int_a^b y dx$, where $y = g(x)$, is a special case of (5.6).

Similarly, if C is represented in the form

$$x = F(y), \quad c \leq y \leq d,$$

and the direction of C is that of increasing y , then

$$\int_C f(x, y) dx = \int_c^d f[F(y), y] F'(y) dy, \quad (5.8)$$

$$\int_C f(x, y) dy = \int_c^d f[F(y), y] dy. \quad (5.9)$$

In most applications the line integrals appear as a combination,

$$\int_C P(x, y) dx + \int_C Q(x, y) dy,$$

which is abbreviated as follows:

$$\int_C [P(x, y) dx + Q(x, y) dy] \quad \text{or} \quad \int_C P(x, y) dx + Q(x, y) dy,$$

the brackets being used only when necessary.

In the formulas thus far the direction of C has been that of increasing parameter. If the opposite direction is chosen, upper and lower limits are reversed on all integrals. Thus (5.4) becomes

$$\int_C f(x, y) dx = \int_k^h f[\phi(t), \psi(t)] \phi'(t) dt. \quad (5.4')$$

The line integral is therefore multiplied by -1 . Often it is convenient to specify the path by its equations in some form and to indicate the direction by using the initial and terminal points as lower and upper limits:

$$\int_A^B P dx + Q dy \quad \text{or} \quad \int_{(x_1, y_1)}^{(x_2, y_2)} P dx + Q dy.$$

It will be seen later that under certain conditions, one needs only prescribe initial and terminal points:

$$\int_A^B P dx + Q dy.$$

EXAMPLE 1 To evaluate

$$\int_C^{(-1,0)}_{(1,0)} (x^3 - y^3) dy,$$

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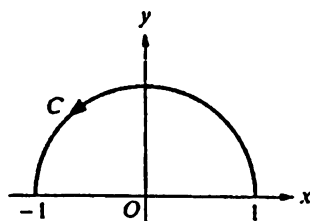


Figure 5.5 Example 1.

where C is the semicircle $y = \sqrt{1 - x^2}$ shown in Fig. 5.5, one can represent C parametrically:

$$x = \cos t, \quad y = \sin t, \quad 0 \leq t \leq \pi,$$

and the integral becomes

$$\int_0^\pi (\cos^3 t - \sin^3 t) \cos t \, dt = \frac{3\pi}{8}.$$

One can use x as parameter, and the integral becomes

$$\int_1^{-1} \left[x^3(1 - x^2)^{\frac{1}{2}} \right] \frac{-x}{\sqrt{1 - x^2}} dx;$$

this is clearly in a more awkward form for integration. The substitution $x = \cos t$ brings one back to the parametric form. One can use y as parameter but has then to split the integral into two parts, from $(1, 0)$ to $(0, 1)$ and from $(0, 1)$ to $(-1, 0)$;

$$\int_0^1 \left[(1 - y^2)^{\frac{1}{2}} - y^3 \right] dy + \int_1^0 \left[-(1 - y^2)^{\frac{1}{2}} - y^3 \right] dy = 2 \int_0^1 (1 - y^2)^{\frac{1}{2}} dy.$$

Note that $x = \sqrt{1 - y^2}$ on the first part of the path and $x = -\sqrt{1 - y^2}$ on the second part. ■

EXAMPLE 2 Let C be the parabolic arc $y = x^2$ from $(0, 0)$ to $(-1, 1)$. Then

$$\int_C xy^2 dx + x^2y dy = \int_0^{-1} \left(xy^2 + x^2y \frac{dy}{dx} \right) dx = \int_0^{-1} (x^5 + 2x^5) dx = \frac{1}{2}. \quad \blacksquare$$

If C is a *closed* curve, then there is no need to specify initial and terminal point, though the direction must be indicated. If C is a simple closed curve (traced just once), then one need only specify which of the two possible directions is chosen. The notations

$$(a) \oint P dx + Q dy, \quad (b) \oint P dx + Q dy$$

refer to the two cases of Figs. 5.6(a) and 5.6(b). The counterclockwise arrow refers to what is roughly a counterclockwise direction on C ; this will be termed the *positive* direction (as for angular measure); the clockwise direction will be called

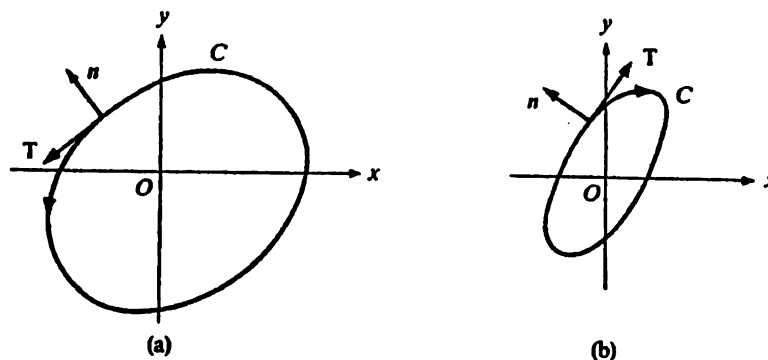


Figure 5.6 (a) Positive direction and (b) negative direction on a simple closed path.

the *negative* direction. It should be noted that the direction can be specified by reference to the unit tangent vector T in the direction of integration and the unit normal vector n that points to the outside of the region bounded by C ; for the positive direction, n is 90° behind T , as in Fig. 5.6(a); for the negative direction, n is 90° ahead of T as in Fig. 5.6(b).

EXAMPLE 3 To evaluate

$$\oint_C y^2 dx + x^2 dy,$$

where C is the triangle with vertices $(1, 0)$, $(1, 1)$, $(0, 0)$, shown in Fig. 5.7, one has to compute three integrals. The first is the integral from $(0, 0)$ to $(1, 0)$; along this path, $y = 0$ and, if x is the parameter, $dy = 0$. Hence the first integral is 0. The second integral is that from $(1, 0)$ to $(1, 1)$; if y is used as parameter, this reduces to

$$\int_0^1 dy = 1,$$

since $dx = 0$. For the third integral, from $(1, 1)$ to $(0, 0)$, x can be used as parameter, so that the integral is

$$\int_1^0 2x^2 dx = -\frac{2}{3},$$

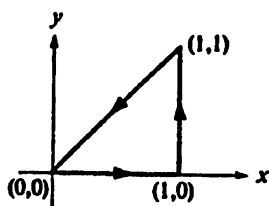


Figure 5.7 Example 3.

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since $dy = dx$. Thus finally

$$\oint_C y^2 dx + x^2 dy = 0 + 1 - \frac{2}{3} = \frac{1}{3}. \quad \blacksquare$$

5.3 INTEGRALS WITH RESPECT TO ARC LENGTH • BASIC PROPERTIES OF LINE INTEGRALS

For a smooth or piecewise smooth path C , as in the preceding section, arc length s is well defined. Thus s can be defined as the distance traversed from the initial point ($t = h$) up to a general t :

$$s = \int_h^t \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} dt. \quad (5.10)$$

If the curve C is directed with increasing t , then s also increases in the direction of motion, going from 0 up to the length L of C . Let C be subdivided as in Fig. 5.4 and let $\Delta_i s$ denote the increment in s from t_{i-1} to t_i , that is, the distance moved in this interval. One then makes the definition

$$\int_C f(x, y) ds = \lim_{\substack{n \rightarrow \infty \\ \max \Delta_i s \rightarrow 0}} \sum_{i=1}^n f(x_i^*, y_i^*) \Delta_i s. \quad (5.11)$$

If f is continuous on C , this integral will exist and can be evaluated as follows:

$$\int_C f(x, y) ds = \int_h^k f[\phi(t), \psi(t)] \sqrt{\phi'(t)^2 + \psi'(t)^2} dt. \quad (5.12)$$

This is proved in the same way as (5.4) and (5.5), with the aid of the formula

$$\frac{ds}{dt} = \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2} = \sqrt{\phi'(t)^2 + \psi'(t)^2}.$$

One can in principle use s itself as the parameter on the curve C ; if this is done, x and y become functions of s : $x = x(s)$, $y = y(s)$. The point $[x(s), y(s)]$ is then the position of the moving point after a distance s has been traversed. In this case, (5.11) reduces to a definite integral with respect to s :

$$\int_C f(x, y) ds = \int_0^L f[x(s), y(s)] ds. \quad (5.13)$$

If x is used as parameter, one has

$$\int_C f(x, y) ds = \int_a^b f[x, y(x)] \sqrt{1 + \left(\frac{dy}{dx}\right)^2} dx; \quad (5.14)$$

there is an analogous formula for y .

The basic combination

$$\int_C P dx + Q dy$$

Nemeth Symbols

⠠	punctuation indicator	⠠⠠	less than sign
⠠⠠	boldface letter indicator	⠠⠠⠠⠠	less than or equal sign
⠠⠠	begin modified expression	⠠⠠	opening parenthesis
⠠⠠	begin modifier below	⠠⠠	closing parenthesis
⠠⠠	begin modifier above	⠠⠠⠠	opening bracket
⠠⠠	end modified expression	⠠⠠⠠	closing bracket
⠠⠠	begin subscript	⠠⠠⠠	asterisk
⠠⠠⠠	begin subscript within subscript	⠠⠠	integral sign
⠠⠠	begin superscript	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	integral sign with superimposed circle bearing counterclockwise arrow
⠠⠠⠠	begin subscript within superscript	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	integral sign with superimposed circle bearing clockwise arrow
⠠⠠	comma within subscript or superscript	⠠⠠⠠⠠⠠⠠⠠⠠⠠⠠	
⠠⠠	return to baseline after subscript or superscript		
⠠⠠	begin square root	⠠⠠⠠	infinity sign
⠠⠠	end square root	⠠⠠	prime sign
⠠⠠	begin fraction	⠠⠠⠠⠠	degree sign
⠠⠠⠠	begin fraction portion of mixed number	⠠⠠⠠	right arrow
⠠⠠	horizontal fraction line	⠠⠠	vertical bar
⠠⠠	end fraction	⠠⠠⠠⠠	solid square
⠠⠠⠠	end fraction portion of mixed number	⠠⠠⠠⠠	Greek capital delta
⠠⠠	mathematical comma	⠠⠠⠠⠠	Greek capital sigma
⠠⠠	decimal point	⠠⠠⠠	Greek alpha
⠠⠠	plus sign	⠠⠠⠠	Greek phi
⠠⠠	minus sign	⠠⠠⠠	Greek pi
⠠⠠	times sign (dot)	⠠⠠⠠	Greek psi
⠠⠠⠠	equal sign	⠠⠠⠠	Greek sigma

UEBC Symbols

⠆	grade one symbol	⠆	decimal point
⠆⠆	grade one word	⠆⠆⠆	ellipsis
⠆⠆⠆	begin grade one passage	⠆⠆	plus sign
⠆	(before a space)	⠆⠆	minus sign
⠆⠆⠆	end grade one passage	⠆⠆⠆	times sign (dot)
⠆⠆⠆⠆⠆⠆	(on a line by itself)	⠆⠆	equal sign
⠆⠆⠆⠆	begin grade one passage	⠆⠆	less than sign
⠆⠆⠆⠆	(on a line by itself)	⠆⠆⠆	less than or equal sign
⠆⠆⠆⠆	end grade one passage	⠆⠆	opening parenthesis
⠆⠆⠆⠆	begin capitalized passage	⠆⠆	closing parenthesis
⠆	(before a space)	⠆⠆	opening bracket
⠆⠆⠆⠆	end capitalized passage	⠆⠆	closing bracket
⠆⠆	italic word	⠆⠆	dash
⠆⠆⠆	begin italic passage	⠆⠆	asterisk
⠆	(before a space)	⠆⠆	integral sign
⠆⠆⠆⠆	end italic passage	⠆⠆⠆	infinity sign
⠆⠆⠆	bold letter	⠆⠆	prime sign
⠆⠆⠆	bold word	⠆⠆⠆	degree sign
⠆⠆	begin fraction	⠆⠆⠆	right arrow
⠆⠆⠆	fraction line	⠆⠆⠆	vertical line
⠆⠆	end fraction	⠆⠆⠆⠆⠆	filled square
⠆	begin compound item	⠆⠆⠆⠆⠆	transcriber-defined shape: circle with counterclockwise arrow
⠆	end compound item	⠆⠆⠆⠆⠆	transcriber-defined shape: circle with clockwise arrow
⠆	begin square root	⠆⠆⠆⠆⠆	end shape
⠆	end square root	⠆⠆⠆⠆⠆	capital Greek delta
⠆	superscript next item	⠆⠆⠆⠆⠆	capital Greek sigma
⠆	subscript next item	⠆⠆⠆	Greek alpha
⠆⠆⠆	next item directly below previous item	⠆⠆⠆	Greek phi
⠆⠆⠆	next item directly above previous item	⠆⠆⠆	Greek pi
⠆	superimpose next symbol on previous symbol	⠆⠆⠆	Greek psi
⠆⠆⠆	arrow over previous item	⠆⠆⠆	Greek sigma

∫ (x^2 + 3x - 5) dx = (1/3)x^3 + (3/2)x^2 - 5x + C

∫ (2x^3 - 4x + 7) dx = (1/2)x^4 - 2x^2 + 7x + C

∫ (x^4 - 2x^2 + 1) dx = (1/5)x^5 - (2/3)x^3 + x + C

∫ (x^3 + 2x^2 - 3x + 4) dx = (1/4)x^4 + (2/3)x^3 - (3/2)x^2 + 4x + C

∫ (5x^2 - 7x + 1) dx = (5/3)x^3 - (7/2)x^2 + x + C

∫ (x^5 + 3x^4 - 2x^3 + 1) dx = (1/6)x^6 + (3/5)x^5 - (1/2)x^4 + x + C

∫ (2x^4 - 5x^3 + 4x^2 - 1) dx = (1/2)x^5 - (5/4)x^4 + (4/3)x^3 - x + C

∫ (x^6 + 3x^5 - 2x^4 + 5x^3 - 1) dx = (1/7)x^7 + (3/6)x^6 - (1/2)x^5 + (5/4)x^4 - x + C

∫ (x^7 + 2x^6 - 3x^5 + 4x^4 - 5x^3 + 6x^2 - 7x + 8) dx = (1/8)x^8 + (1/3)x^7 - (3/4)x^6 + x^5 - (5/4)x^4 + 2x^3 - (7/2)x^2 + 4x + C

∫ (x^8 + 3x^7 - 2x^6 + 5x^5 - 4x^4 + 3x^3 - 2x^2 + 1) dx = (1/9)x^9 + (1/2)x^8 - (1/3)x^7 + (1/2)x^6 - x^5 + (3/4)x^4 - (2/3)x^3 + (2/5)x^2 + x + C

∫ (x^9 + 2x^8 - 3x^7 + 4x^6 - 5x^5 + 6x^4 - 7x^3 + 8x^2 - 9x + 10) dx = (1/10)x^10 + (1/4)x^9 - (3/5)x^8 + (2/3)x^7 - x^6 + (3/2)x^5 - (7/4)x^4 + (2/3)x^3 - (9/5)x^2 + 5x + C

∫ (x^10 + 3x^9 - 2x^8 + 5x^7 - 4x^6 + 3x^5 - 2x^4 + 1) dx = (1/11)x^11 + (3/10)x^10 - (1/4)x^9 + (1/2)x^8 - (2/3)x^7 + (2/5)x^6 - (2/4)x^5 + (1/3)x^4 + x + C

∫ (x^11 + 2x^10 - 3x^9 + 4x^8 - 5x^7 + 6x^6 - 7x^5 + 8x^4 - 9x^3 + 10x^2 - 11x + 12) dx = (1/12)x^12 + (1/5)x^11 - (3/10)x^10 + (2/3)x^9 - (5/6)x^8 + (1/2)x^7 - (7/10)x^6 + (2/3)x^5 - (9/4)x^4 + (5/2)x^3 - (11/2)x^2 + 6x + C

∫ (x^12 + 3x^11 - 2x^10 + 5x^9 - 4x^8 + 3x^7 - 2x^6 + 1) dx = (1/13)x^13 + (3/12)x^12 - (1/5)x^11 + (1/2)x^10 - (2/3)x^9 + (3/4)x^8 - (1/3)x^7 + (2/5)x^6 + (1/12)x^5 + x + C

∫ (x^13 + 2x^12 - 3x^11 + 4x^10 - 5x^9 + 6x^8 - 7x^7 + 8x^6 - 9x^5 + 10x^4 - 11x^3 + 12x^2 - 13x + 14) dx = (1/14)x^14 + (1/6)x^13 - (3/10)x^12 + (2/3)x^11 - (5/12)x^10 + (1/2)x^9 - (7/10)x^8 + (2/3)x^7 - (9/4)x^6 + (5/2)x^5 - (11/2)x^4 + 6x^3 - (13/2)x^2 + 7x + C

∫ (x^14 + 3x^13 - 2x^12 + 5x^11 - 4x^10 + 3x^9 - 2x^8 + 1) dx = (1/15)x^15 + (3/12)x^14 - (1/5)x^13 + (1/2)x^12 - (2/3)x^11 + (3/4)x^10 - (1/3)x^9 + (2/5)x^8 + (1/15)x^7 + x + C

∫ (x^15 + 2x^14 - 3x^13 + 4x^12 - 5x^11 + 6x^10 - 7x^9 + 8x^8 - 9x^7 + 10x^6 - 11x^5 + 12x^4 - 13x^3 + 14x^2 - 15x + 16) dx = (1/16)x^16 + (1/7)x^15 - (3/10)x^14 + (2/3)x^13 - (5/12)x^12 + (1/2)x^11 - (7/10)x^10 + (2/3)x^9 - (9/4)x^8 + (5/2)x^7 - (11/2)x^6 + 6x^5 - (13/2)x^4 + 7x^3 - (15/2)x^2 + 8x + C

∫ (x^16 + 3x^15 - 2x^14 + 5x^13 - 4x^12 + 3x^11 - 2x^10 + 1) dx = (1/17)x^17 + (3/12)x^16 - (1/5)x^15 + (1/2)x^14 - (2/3)x^13 + (3/4)x^12 - (1/3)x^11 + (2/5)x^10 + (1/17)x^9 + x + C

∫ (x^17 + 2x^16 - 3x^15 + 4x^14 - 5x^13 + 6x^12 - 7x^11 + 8x^10 - 9x^9 + 10x^8 - 11x^7 + 12x^6 - 13x^5 + 14x^4 - 15x^3 + 16x^2 - 17x + 18) dx = (1/18)x^18 + (1/8)x^17 - (3/10)x^16 + (2/3)x^15 - (5/12)x^14 + (1/2)x^13 - (7/10)x^12 + (2/3)x^11 - (9/4)x^10 + (5/2)x^9 - (11/2)x^8 + 6x^7 - (13/2)x^6 + 7x^5 - (15/2)x^4 + 8x^3 - (17/2)x^2 + 9x + C

∫ (x^18 + 3x^17 - 2x^16 + 5x^15 - 4x^14 + 3x^13 - 2x^12 + 1) dx = (1/19)x^19 + (3/12)x^18 - (1/5)x^17 + (1/2)x^16 - (2/3)x^15 + (3/4)x^14 - (1/3)x^13 + (2/5)x^12 + (1/19)x^11 + x + C

Calculus I
Chapter 10
Section 10.1

Example 1: Find the area of the region bounded by the parabola $y = -x^2 + 4$ and the x-axis.

Solution: The parabola $y = -x^2 + 4$ opens downwards with vertex at $(0, 4)$. It intersects the x-axis at $(-2, 0)$ and $(2, 0)$.

The region is symmetric about the y-axis. The area can be found by integrating from $x = -2$ to $x = 2$:

$$A = \int_{-2}^2 (-x^2 + 4) dx$$

$$= \left[-\frac{1}{3}x^3 + 4x \right]_{-2}^2$$

$$= \left(-\frac{8}{3} + 8 \right) - \left(\frac{8}{3} - 8 \right)$$

$$= \frac{16}{3} - \frac{8}{3} + 8 - \frac{8}{3} + 8$$

$$= \frac{16}{3} + \frac{16}{3} = \frac{32}{3}$$

The area of the region is $\frac{32}{3}$ square units.

Example 2: Find the area of the region bounded by the lines $y = x$, $y = 2x$, and $x = 2$.

Solution: The region is bounded by the lines $y = x$, $y = 2x$, and $x = 2$. The vertices are $(0, 0)$, $(2, 2)$, and $(2, 4)$.

The area can be found by integrating with respect to x from $x = 0$ to $x = 2$:

$$A = \int_0^2 (2x - x) dx$$

$$= \int_0^2 x dx$$

$$= \left[\frac{1}{2}x^2 \right]_0^2$$

$$= \frac{1}{2}(4) - 0 = 2$$

The area of the region is 2 square units.

Example 3: Find the area of the region bounded by the parabola $y = x^2$ and the line $y = 2 - x^2$.

Solution: The parabola $y = x^2$ opens upwards with vertex at $(0, 0)$. The parabola $y = 2 - x^2$ opens downwards with vertex at $(0, 2)$. They intersect at $(-1, 1)$ and $(1, 1)$.

The region is symmetric about the y-axis. The area can be found by integrating from $x = -1$ to $x = 1$:

$$A = \int_{-1}^1 (2 - x^2 - x^2) dx$$

$$= \int_{-1}^1 (2 - 2x^2) dx$$

$$= 2 \int_{-1}^1 (1 - x^2) dx$$

$$= 2 \left[x - \frac{1}{3}x^3 \right]_{-1}^1$$

$$= 2 \left(\left(1 - \frac{1}{3} \right) - \left(-1 + \frac{1}{3} \right) \right)$$

$$= 2 \left(\frac{2}{3} + \frac{2}{3} \right) = 2 \left(\frac{4}{3} \right) = \frac{8}{3}$$

The area of the region is $\frac{8}{3}$ square units.

Example 4: Find the area of the region bounded by the curves $y = \sqrt{x}$ and $y = x^2$.

Solution: The curve $y = \sqrt{x}$ is the upper half of the parabola $y^2 = x$ with vertex at $(0, 0)$. The curve $y = x^2$ is the parabola opening upwards with vertex at $(0, 0)$. They intersect at $(1, 1)$ and $(0, 0)$.

The region is bounded by the curves $y = \sqrt{x}$ and $y = x^2$ from $x = 0$ to $x = 1$:

$$A = \int_0^1 (\sqrt{x} - x^2) dx$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{1}{3}x^3 \right]_0^1$$

$$= \left(\frac{2}{3} - \frac{1}{3} \right) - 0 = \frac{1}{3}$$

The area of the region is $\frac{1}{3}$ square units.

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Calculus is a branch of mathematics that deals with the study of change. It is divided into two main parts: differential calculus and integral calculus. Differential calculus is concerned with the study of rates of change and slopes of curves. Integral calculus is concerned with the study of accumulation of quantities and areas under curves. The two parts are related by the fundamental theorem of calculus, which states that the derivative of an integral is the original function, and the integral of a derivative is the original function plus a constant. Calculus is used in many fields, including physics, engineering, economics, and biology. It is a powerful tool for understanding the world around us.

$\frac{1}{x^2} = x^{-2}$
 $\frac{d}{dx} x^{-2} = -2x^{-3} = -\frac{2}{x^3}$

$\frac{1}{x^3} = x^{-3}$
 $\frac{d}{dx} x^{-3} = -3x^{-4} = -\frac{3}{x^4}$

$\frac{1}{x^4} = x^{-4}$
 $\frac{d}{dx} x^{-4} = -4x^{-5} = -\frac{4}{x^5}$

$\frac{1}{x^5} = x^{-5}$
 $\frac{d}{dx} x^{-5} = -5x^{-6} = -\frac{5}{x^6}$

$\frac{1}{x^6} = x^{-6}$
 $\frac{d}{dx} x^{-6} = -6x^{-7} = -\frac{6}{x^7}$

$\frac{1}{x^7} = x^{-7}$
 $\frac{d}{dx} x^{-7} = -7x^{-8} = -\frac{7}{x^8}$

$\frac{1}{x^8} = x^{-8}$
 $\frac{d}{dx} x^{-8} = -8x^{-9} = -\frac{8}{x^9}$

$\frac{1}{x^9} = x^{-9}$
 $\frac{d}{dx} x^{-9} = -9x^{-10} = -\frac{9}{x^{10}}$

$\frac{1}{x^{10}} = x^{-10}$
 $\frac{d}{dx} x^{-10} = -10x^{-11} = -\frac{10}{x^{11}}$

$\frac{1}{x^{11}} = x^{-11}$
 $\frac{d}{dx} x^{-11} = -11x^{-12} = -\frac{11}{x^{12}}$

$\frac{1}{x^{12}} = x^{-12}$
 $\frac{d}{dx} x^{-12} = -12x^{-13} = -\frac{12}{x^{13}}$

$\frac{1}{x^{13}} = x^{-13}$
 $\frac{d}{dx} x^{-13} = -13x^{-14} = -\frac{13}{x^{14}}$

$\frac{1}{x^{14}} = x^{-14}$
 $\frac{d}{dx} x^{-14} = -14x^{-15} = -\frac{14}{x^{15}}$

$\frac{1}{x^{15}} = x^{-15}$
 $\frac{d}{dx} x^{-15} = -15x^{-16} = -\frac{15}{x^{16}}$

$\frac{1}{x^{16}} = x^{-16}$
 $\frac{d}{dx} x^{-16} = -16x^{-17} = -\frac{16}{x^{17}}$

$\frac{1}{x^{17}} = x^{-17}$
 $\frac{d}{dx} x^{-17} = -17x^{-18} = -\frac{17}{x^{18}}$

$\frac{1}{x^{18}} = x^{-18}$
 $\frac{d}{dx} x^{-18} = -18x^{-19} = -\frac{18}{x^{19}}$

$\frac{1}{x^{19}} = x^{-19}$
 $\frac{d}{dx} x^{-19} = -19x^{-20} = -\frac{19}{x^{20}}$

$\frac{1}{x^{20}} = x^{-20}$
 $\frac{d}{dx} x^{-20} = -20x^{-21} = -\frac{20}{x^{21}}$

$$\frac{d}{dx} \int_0^x \sin t \, dt = \sin x$$

$$\frac{d}{dx} \int_0^x \cos t \, dt = \cos x$$

$$\frac{d}{dx} \int_0^x e^t \, dt = e^x$$

$$\frac{d}{dx} \int_0^x \ln t \, dt = \ln x$$

$$\frac{d}{dx} \int_0^x \frac{1}{t} \, dt = \frac{1}{x}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^2} \, dt = -\frac{1}{x^2}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^3} \, dt = -\frac{1}{2x^2}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^4} \, dt = -\frac{1}{3x^3}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^5} \, dt = -\frac{1}{4x^4}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^6} \, dt = -\frac{1}{5x^5}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^7} \, dt = -\frac{1}{6x^6}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^8} \, dt = -\frac{1}{7x^7}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^9} \, dt = -\frac{1}{8x^8}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{10}} \, dt = -\frac{1}{9x^9}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{11}} \, dt = -\frac{1}{10x^{10}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{12}} \, dt = -\frac{1}{11x^{11}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{13}} \, dt = -\frac{1}{12x^{12}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{14}} \, dt = -\frac{1}{13x^{13}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{15}} \, dt = -\frac{1}{14x^{14}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{16}} \, dt = -\frac{1}{15x^{15}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{17}} \, dt = -\frac{1}{16x^{16}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{18}} \, dt = -\frac{1}{17x^{17}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{19}} \, dt = -\frac{1}{18x^{18}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{20}} \, dt = -\frac{1}{19x^{19}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{21}} \, dt = -\frac{1}{20x^{20}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{22}} \, dt = -\frac{1}{21x^{21}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{23}} \, dt = -\frac{1}{22x^{22}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{24}} \, dt = -\frac{1}{23x^{23}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{25}} \, dt = -\frac{1}{24x^{24}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{26}} \, dt = -\frac{1}{25x^{25}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{27}} \, dt = -\frac{1}{26x^{26}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{28}} \, dt = -\frac{1}{27x^{27}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{29}} \, dt = -\frac{1}{28x^{28}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{30}} \, dt = -\frac{1}{29x^{29}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{31}} \, dt = -\frac{1}{30x^{30}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{32}} \, dt = -\frac{1}{31x^{31}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{33}} \, dt = -\frac{1}{32x^{32}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{34}} \, dt = -\frac{1}{33x^{33}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{35}} \, dt = -\frac{1}{34x^{34}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{36}} \, dt = -\frac{1}{35x^{35}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{37}} \, dt = -\frac{1}{36x^{36}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{38}} \, dt = -\frac{1}{37x^{37}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{39}} \, dt = -\frac{1}{38x^{38}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{40}} \, dt = -\frac{1}{39x^{39}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{41}} \, dt = -\frac{1}{40x^{40}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{42}} \, dt = -\frac{1}{41x^{41}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{43}} \, dt = -\frac{1}{42x^{42}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{44}} \, dt = -\frac{1}{43x^{43}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{45}} \, dt = -\frac{1}{44x^{44}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{46}} \, dt = -\frac{1}{45x^{45}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{47}} \, dt = -\frac{1}{46x^{46}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{48}} \, dt = -\frac{1}{47x^{47}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{49}} \, dt = -\frac{1}{48x^{48}}$$

$$\frac{d}{dx} \int_0^x \frac{1}{t^{50}} \, dt = -\frac{1}{49x^{49}}$$

Braille representation of the text on page 144 of Sample 3. Calculus. The page contains a series of paragraphs of text, including mathematical concepts and definitions, rendered in Braille.

Let $f(x) = \sin(x)$ and $g(x) = \cos(x)$. Then $f'(x) = \cos(x)$ and $g'(x) = -\sin(x)$.
Using the product rule, $(fg)' = f'g + fg'$.
So $(\sin(x)\cos(x))' = \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x)$.
This is the double angle formula for cosine: $\cos(2x) = \cos^2(x) - \sin^2(x)$.
Therefore, $(\sin(x)\cos(x))' = \cos(2x)$.
Now, let $h(x) = \sin^2(x)$. Then $h'(x) = 2\sin(x)\cos(x) = \sin(2x)$.
Similarly, $(\cos^2(x))' = -2\cos(x)\sin(x) = -\sin(2x)$.
Thus, $(\sin^2(x) - \cos^2(x))' = \sin(2x) - (-\sin(2x)) = 2\sin(2x)$.
This is the double angle formula for sine: $\sin(2x) = 2\sin(x)\cos(x)$.
Therefore, $(\sin^2(x) - \cos^2(x))' = 2\sin(2x)$.

$$\begin{aligned} \frac{d}{dx}(\sin^2(x) - \cos^2(x)) &= 2\sin(x)\cos(x) - (-2\cos(x)\sin(x)) \\ &= 2\sin(x)\cos(x) + 2\cos(x)\sin(x) \\ &= 4\sin(x)\cos(x) = 2\sin(2x) \end{aligned}$$

Let $f(x) = \sin(x)$ and $g(x) = \cos(x)$. Then $f'(x) = \cos(x)$ and $g'(x) = -\sin(x)$.
Using the product rule, $(fg)' = f'g + fg'$.
So $(\sin(x)\cos(x))' = \cos(x)\cos(x) + \sin(x)(-\sin(x)) = \cos^2(x) - \sin^2(x)$.
This is the double angle formula for cosine: $\cos(2x) = \cos^2(x) - \sin^2(x)$.
Therefore, $(\sin(x)\cos(x))' = \cos(2x)$.
Now, let $h(x) = \sin^2(x)$. Then $h'(x) = 2\sin(x)\cos(x) = \sin(2x)$.
Similarly, $(\cos^2(x))' = -2\cos(x)\sin(x) = -\sin(2x)$.
Thus, $(\sin^2(x) - \cos^2(x))' = \sin(2x) - (-\sin(2x)) = 2\sin(2x)$.
This is the double angle formula for sine: $\sin(2x) = 2\sin(x)\cos(x)$.
Therefore, $(\sin^2(x) - \cos^2(x))' = 2\sin(2x)$.

1. $f(x) = \frac{1}{x}$ and $g(x) = \frac{1}{x^2}$. Find $(f+g)(x)$ and $(fg)(x)$.
 $(f+g)(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$
 $(fg)(x) = \frac{1}{x} \cdot \frac{1}{x^2} = \frac{1}{x^3}$

$(f-g)(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

$(fg)(x) = \frac{1}{x^3}$

$(f/g)(x) = \frac{1/x}{1/x^2} = x$

$(f \cdot g)(x) = \frac{1}{x} \cdot \frac{1}{x^2} = \frac{1}{x^3}$

$(f+g)(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$

$(f-g)(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

$(fg)(x) = \frac{1}{x^3}$

$(f/g)(x) = x$

$(fg)(x) = \frac{1}{x^3}$

$(f+g)(x) = \frac{1}{x} + \frac{1}{x^2} = \frac{x+1}{x^2}$

$(f-g)(x) = \frac{1}{x} - \frac{1}{x^2} = \frac{x-1}{x^2}$

$(fg)(x) = \frac{1}{x^3}$

$(f/g)(x) = x$

$(fg)(x) = \frac{1}{x^3}$

1. The function $f(x) = x^2 - 3x + 4$ has a minimum value of $\frac{25}{4}$ at $x = \frac{3}{2}$.
2. The function $f(x) = x^2 - 6x + 9$ has a minimum value of 0 at $x = 3$.
3. The function $f(x) = x^2 - 4x + 4$ has a minimum value of 0 at $x = 2$.
4. The function $f(x) = x^2 - 2x + 1$ has a minimum value of 0 at $x = 1$.
5. The function $f(x) = x^2 - 8x + 16$ has a minimum value of 0 at $x = 4$.
6. The function $f(x) = x^2 - 10x + 25$ has a minimum value of 0 at $x = 5$.
7. The function $f(x) = x^2 - 12x + 36$ has a minimum value of 0 at $x = 6$.
8. The function $f(x) = x^2 - 14x + 49$ has a minimum value of 0 at $x = 7$.
9. The function $f(x) = x^2 - 16x + 64$ has a minimum value of 0 at $x = 8$.
10. The function $f(x) = x^2 - 18x + 81$ has a minimum value of 0 at $x = 9$.
11. The function $f(x) = x^2 - 20x + 100$ has a minimum value of 0 at $x = 10$.
12. The function $f(x) = x^2 - 22x + 121$ has a minimum value of 0 at $x = 11$.
13. The function $f(x) = x^2 - 24x + 144$ has a minimum value of 0 at $x = 12$.
14. The function $f(x) = x^2 - 26x + 169$ has a minimum value of 0 at $x = 13$.
15. The function $f(x) = x^2 - 28x + 196$ has a minimum value of 0 at $x = 14$.
16. The function $f(x) = x^2 - 30x + 225$ has a minimum value of 0 at $x = 15$.
17. The function $f(x) = x^2 - 32x + 256$ has a minimum value of 0 at $x = 16$.
18. The function $f(x) = x^2 - 34x + 289$ has a minimum value of 0 at $x = 17$.
19. The function $f(x) = x^2 - 36x + 324$ has a minimum value of 0 at $x = 18$.
20. The function $f(x) = x^2 - 38x + 361$ has a minimum value of 0 at $x = 19$.
21. The function $f(x) = x^2 - 40x + 400$ has a minimum value of 0 at $x = 20$.
22. The function $f(x) = x^2 - 42x + 441$ has a minimum value of 0 at $x = 21$.
23. The function $f(x) = x^2 - 44x + 484$ has a minimum value of 0 at $x = 22$.
24. The function $f(x) = x^2 - 46x + 529$ has a minimum value of 0 at $x = 23$.
25. The function $f(x) = x^2 - 48x + 576$ has a minimum value of 0 at $x = 24$.
26. The function $f(x) = x^2 - 50x + 625$ has a minimum value of 0 at $x = 25$.
27. The function $f(x) = x^2 - 52x + 676$ has a minimum value of 0 at $x = 26$.
28. The function $f(x) = x^2 - 54x + 729$ has a minimum value of 0 at $x = 27$.
29. The function $f(x) = x^2 - 56x + 784$ has a minimum value of 0 at $x = 28$.
30. The function $f(x) = x^2 - 58x + 841$ has a minimum value of 0 at $x = 29$.
31. The function $f(x) = x^2 - 60x + 900$ has a minimum value of 0 at $x = 30$.
32. The function $f(x) = x^2 - 62x + 961$ has a minimum value of 0 at $x = 31$.
33. The function $f(x) = x^2 - 64x + 1024$ has a minimum value of 0 at $x = 32$.
34. The function $f(x) = x^2 - 66x + 1089$ has a minimum value of 0 at $x = 33$.
35. The function $f(x) = x^2 - 68x + 1156$ has a minimum value of 0 at $x = 34$.
36. The function $f(x) = x^2 - 70x + 1225$ has a minimum value of 0 at $x = 35$.
37. The function $f(x) = x^2 - 72x + 1296$ has a minimum value of 0 at $x = 36$.
38. The function $f(x) = x^2 - 74x + 1369$ has a minimum value of 0 at $x = 37$.
39. The function $f(x) = x^2 - 76x + 1444$ has a minimum value of 0 at $x = 38$.
40. The function $f(x) = x^2 - 78x + 1521$ has a minimum value of 0 at $x = 39$.
41. The function $f(x) = x^2 - 80x + 1600$ has a minimum value of 0 at $x = 40$.
42. The function $f(x) = x^2 - 82x + 1681$ has a minimum value of 0 at $x = 41$.
43. The function $f(x) = x^2 - 84x + 1764$ has a minimum value of 0 at $x = 42$.
44. The function $f(x) = x^2 - 86x + 1849$ has a minimum value of 0 at $x = 43$.
45. The function $f(x) = x^2 - 88x + 1936$ has a minimum value of 0 at $x = 44$.
46. The function $f(x) = x^2 - 90x + 2025$ has a minimum value of 0 at $x = 45$.
47. The function $f(x) = x^2 - 92x + 2116$ has a minimum value of 0 at $x = 46$.
48. The function $f(x) = x^2 - 94x + 2209$ has a minimum value of 0 at $x = 47$.
49. The function $f(x) = x^2 - 96x + 2304$ has a minimum value of 0 at $x = 48$.
50. The function $f(x) = x^2 - 98x + 2401$ has a minimum value of 0 at $x = 49$.
51. The function $f(x) = x^2 - 100x + 2500$ has a minimum value of 0 at $x = 50$.
52. The function $f(x) = x^2 - 102x + 2601$ has a minimum value of 0 at $x = 51$.
53. The function $f(x) = x^2 - 104x + 2704$ has a minimum value of 0 at $x = 52$.
54. The function $f(x) = x^2 - 106x + 2809$ has a minimum value of 0 at $x = 53$.
55. The function $f(x) = x^2 - 108x + 2916$ has a minimum value of 0 at $x = 54$.
56. The function $f(x) = x^2 - 110x + 3025$ has a minimum value of 0 at $x = 55$.
57. The function $f(x) = x^2 - 112x + 3136$ has a minimum value of 0 at $x = 56$.
58. The function $f(x) = x^2 - 114x + 3249$ has a minimum value of 0 at $x = 57$.
59. The function $f(x) = x^2 - 116x + 3364$ has a minimum value of 0 at $x = 58$.
60. The function $f(x) = x^2 - 118x + 3481$ has a minimum value of 0 at $x = 59$.
61. The function $f(x) = x^2 - 120x + 3600$ has a minimum value of 0 at $x = 60$.
62. The function $f(x) = x^2 - 122x + 3721$ has a minimum value of 0 at $x = 61$.
63. The function $f(x) = x^2 - 124x + 3844$ has a minimum value of 0 at $x = 62$.
64. The function $f(x) = x^2 - 126x + 3969$ has a minimum value of 0 at $x = 63$.
65. The function $f(x) = x^2 - 128x + 4096$ has a minimum value of 0 at $x = 64$.
66. The function $f(x) = x^2 - 130x + 4225$ has a minimum value of 0 at $x = 65$.
67. The function $f(x) = x^2 - 132x + 4356$ has a minimum value of 0 at $x = 66$.
68. The function $f(x) = x^2 - 134x + 4489$ has a minimum value of 0 at $x = 67$.
69. The function $f(x) = x^2 - 136x + 4624$ has a minimum value of 0 at $x = 68$.
70. The function $f(x) = x^2 - 138x + 4761$ has a minimum value of 0 at $x = 69$.
71. The function $f(x) = x^2 - 140x + 4900$ has a minimum value of 0 at $x = 70$.
72. The function $f(x) = x^2 - 142x + 5041$ has a minimum value of 0 at $x = 71$.
73. The function $f(x) = x^2 - 144x + 5184$ has a minimum value of 0 at $x = 72$.
74. The function $f(x) = x^2 - 146x + 5329$ has a minimum value of 0 at $x = 73$.
75. The function $f(x) = x^2 - 148x + 5476$ has a minimum value of 0 at $x = 74$.
76. The function $f(x) = x^2 - 150x + 5625$ has a minimum value of 0 at $x = 75$.
77. The function $f(x) = x^2 - 152x + 5776$ has a minimum value of 0 at $x = 76$.
78. The function $f(x) = x^2 - 154x + 5929$ has a minimum value of 0 at $x = 77$.
79. The function $f(x) = x^2 - 156x + 6084$ has a minimum value of 0 at $x = 78$.
80. The function $f(x) = x^2 - 158x + 6241$ has a minimum value of 0 at $x = 79$.
81. The function $f(x) = x^2 - 160x + 6400$ has a minimum value of 0 at $x = 80$.
82. The function $f(x) = x^2 - 162x + 6561$ has a minimum value of 0 at $x = 81$.
83. The function $f(x) = x^2 - 164x + 6724$ has a minimum value of 0 at $x = 82$.
84. The function $f(x) = x^2 - 166x + 6889$ has a minimum value of 0 at $x = 83$.
85. The function $f(x) = x^2 - 168x + 7056$ has a minimum value of 0 at $x = 84$.
86. The function $f(x) = x^2 - 170x + 7225$ has a minimum value of 0 at $x = 85$.
87. The function $f(x) = x^2 - 172x + 7396$ has a minimum value of 0 at $x = 86$.
88. The function $f(x) = x^2 - 174x + 7569$ has a minimum value of 0 at $x = 87$.
89. The function $f(x) = x^2 - 176x + 7744$ has a minimum value of 0 at $x = 88$.
90. The function $f(x) = x^2 - 178x + 7921$ has a minimum value of 0 at $x = 89$.
91. The function $f(x) = x^2 - 180x + 8100$ has a minimum value of 0 at $x = 90$.
92. The function $f(x) = x^2 - 182x + 8281$ has a minimum value of 0 at $x = 91$.
93. The function $f(x) = x^2 - 184x + 8464$ has a minimum value of 0 at $x = 92$.
94. The function $f(x) = x^2 - 186x + 8649$ has a minimum value of 0 at $x = 93$.
95. The function $f(x) = x^2 - 188x + 8836$ has a minimum value of 0 at $x = 94$.
96. The function $f(x) = x^2 - 190x + 9025$ has a minimum value of 0 at $x = 95$.
97. The function $f(x) = x^2 - 192x + 9216$ has a minimum value of 0 at $x = 96$.
98. The function $f(x) = x^2 - 194x + 9409$ has a minimum value of 0 at $x = 97$.
99. The function $f(x) = x^2 - 196x + 9604$ has a minimum value of 0 at $x = 98$.
100. The function $f(x) = x^2 - 198x + 9801$ has a minimum value of 0 at $x = 99$.
101. The function $f(x) = x^2 - 200x + 10000$ has a minimum value of 0 at $x = 100$.

$\frac{1}{x^2} = x^{-2}$

$\frac{d}{dx} x^{-2} = -2x^{-3}$

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Calculus is a branch of mathematics that deals with the study of change. It is divided into two main parts: differential calculus and integral calculus. Differential calculus is concerned with the study of rates of change and slopes of curves. Integral calculus is concerned with the study of accumulation of quantities and the area under a curve.

The derivative of a function $f(x)$ at a point x is denoted by $f'(x)$. It represents the instantaneous rate of change of the function at that point. The derivative of a function $f(x)$ with respect to x is denoted by $\frac{df}{dx}$.

The integral of a function $f(x)$ with respect to x is denoted by $\int f(x) dx$. It represents the total accumulation of the function over a certain interval.

The fundamental theorem of calculus states that the derivative of the integral of a function $f(x)$ with respect to x is equal to the function $f(x)$ itself. In other words, differentiation and integration are inverse processes.

Calculus is a powerful tool that is used in many fields of science and engineering. It is used to model and analyze the behavior of systems that change over time. It is also used to optimize the design of structures and machines.

Calculus is a beautiful and challenging subject. It is a subject that has fascinated mathematicians for centuries. It is a subject that has led to many important discoveries in science and technology.

∫₀¹ (x² + 3x + 2) dx = [$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$]₀¹ = $\frac{1}{3} + \frac{3}{2} + 2 = \frac{17}{6}$

∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

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∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

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∫₀¹ x dx = [1/2 x²]₀¹ = 1/2 (1² - 0²) = 1/2

∫₀¹ x² dx = [1/3 x³]₀¹ = 1/3 (1³ - 0³) = 1/3

∫₀¹ x³ dx = [1/4 x⁴]₀¹ = 1/4 (1⁴ - 0⁴) = 1/4

∫₀¹ xⁿ dx = [1/(n+1) xⁿ⁺¹]₀¹ = 1/(n+1) (1ⁿ⁺¹ - 0ⁿ⁺¹) = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)

∫₀¹ x dx = 1/2

∫₀¹ x² dx = 1/3

∫₀¹ x³ dx = 1/4

∫₀¹ xⁿ dx = 1/(n+1)



$2 \times 2 = 4$ $3 \times 3 = 9$ $4 \times 4 = 16$ $5 \times 5 = 25$ $6 \times 6 = 36$ $7 \times 7 = 49$ $8 \times 8 = 64$ $9 \times 9 = 81$
 $10 \times 10 = 100$ $11 \times 11 = 121$ $12 \times 12 = 144$ $13 \times 13 = 169$ $14 \times 14 = 196$ $15 \times 15 = 225$ $16 \times 16 = 256$ $17 \times 17 = 289$
 $18 \times 18 = 324$ $19 \times 19 = 361$ $20 \times 20 = 400$ $21 \times 21 = 441$ $22 \times 22 = 484$ $23 \times 23 = 529$ $24 \times 24 = 576$ $25 \times 25 = 625$
 $26 \times 26 = 676$ $27 \times 27 = 729$ $28 \times 28 = 784$ $29 \times 29 = 841$ $30 \times 30 = 900$ $31 \times 31 = 961$ $32 \times 32 = 1024$ $33 \times 33 = 1089$
 $34 \times 34 = 1156$ $35 \times 35 = 1225$ $36 \times 36 = 1296$ $37 \times 37 = 1369$ $38 \times 38 = 1444$ $39 \times 39 = 1521$ $40 \times 40 = 1600$ $41 \times 41 = 1681$
 $42 \times 42 = 1764$ $43 \times 43 = 1849$ $44 \times 44 = 1936$ $45 \times 45 = 2025$ $46 \times 46 = 2116$ $47 \times 47 = 2209$ $48 \times 48 = 2304$ $49 \times 49 = 2401$
 $50 \times 50 = 2500$ $51 \times 51 = 2601$ $52 \times 52 = 2704$ $53 \times 53 = 2809$ $54 \times 54 = 2916$ $55 \times 55 = 3025$ $56 \times 56 = 3136$ $57 \times 57 = 3249$
 $58 \times 58 = 3364$ $59 \times 59 = 3481$ $60 \times 60 = 3600$ $61 \times 61 = 3721$ $62 \times 62 = 3844$ $63 \times 63 = 3969$ $64 \times 64 = 4096$ $65 \times 65 = 4225$
 $66 \times 66 = 4356$ $67 \times 67 = 4489$ $68 \times 68 = 4624$ $69 \times 69 = 4761$ $70 \times 70 = 4900$ $71 \times 71 = 5041$ $72 \times 72 = 5184$ $73 \times 73 = 5329$
 $74 \times 74 = 5476$ $75 \times 75 = 5625$ $76 \times 76 = 5776$ $77 \times 77 = 5929$ $78 \times 78 = 6084$ $79 \times 79 = 6241$ $80 \times 80 = 6400$ $81 \times 81 = 6561$
 $82 \times 82 = 6724$ $83 \times 83 = 6889$ $84 \times 84 = 7056$ $85 \times 85 = 7225$ $86 \times 86 = 7396$ $87 \times 87 = 7569$ $88 \times 88 = 7744$ $89 \times 89 = 7921$
 $90 \times 90 = 8100$ $91 \times 91 = 8281$ $92 \times 92 = 8464$ $93 \times 93 = 8649$ $94 \times 94 = 8836$ $95 \times 95 = 9025$ $96 \times 96 = 9216$ $97 \times 97 = 9409$
 $98 \times 98 = 9616$ $99 \times 99 = 9801$ $100 \times 100 = 10000$

$101 \times 101 = 10201$ $102 \times 102 = 10404$ $103 \times 103 = 10609$ $104 \times 104 = 10816$ $105 \times 105 = 11025$
 $106 \times 106 = 11236$ $107 \times 107 = 11449$ $108 \times 108 = 11664$ $109 \times 109 = 11881$ $110 \times 110 = 12100$
 $111 \times 111 = 12321$ $112 \times 112 = 12544$ $113 \times 113 = 12769$ $114 \times 114 = 12996$ $115 \times 115 = 13225$
 $116 \times 116 = 13456$ $117 \times 117 = 13689$ $118 \times 118 = 13924$ $119 \times 119 = 14161$ $120 \times 120 = 14400$
 $121 \times 121 = 14641$ $122 \times 122 = 14884$ $123 \times 123 = 15129$ $124 \times 124 = 15376$ $125 \times 125 = 15625$
 $126 \times 126 = 15876$ $127 \times 127 = 16129$ $128 \times 128 = 16384$ $129 \times 129 = 16641$ $130 \times 130 = 16900$
 $131 \times 131 = 17161$ $132 \times 132 = 17424$ $133 \times 133 = 17689$ $134 \times 134 = 17956$ $135 \times 135 = 18225$
 $136 \times 136 = 18496$ $137 \times 137 = 18769$ $138 \times 138 = 19044$ $139 \times 139 = 19321$ $140 \times 140 = 19600$
 $141 \times 141 = 19881$ $142 \times 142 = 20164$ $143 \times 143 = 20449$ $144 \times 144 = 20736$ $145 \times 145 = 21025$
 $146 \times 146 = 21316$ $147 \times 147 = 21609$ $148 \times 148 = 21904$ $149 \times 149 = 22201$ $150 \times 150 = 22500$
 $151 \times 151 = 22801$ $152 \times 152 = 23104$ $153 \times 153 = 23409$ $154 \times 154 = 23716$ $155 \times 155 = 24025$
 $156 \times 156 = 24336$ $157 \times 157 = 24649$ $158 \times 158 = 24964$ $159 \times 159 = 25281$ $160 \times 160 = 25600$
 $161 \times 161 = 25921$ $162 \times 162 = 26244$ $163 \times 163 = 26569$ $164 \times 164 = 26896$ $165 \times 165 = 27225$
 $166 \times 166 = 27556$ $167 \times 167 = 27889$ $168 \times 168 = 28224$ $169 \times 169 = 28561$ $170 \times 170 = 28900$
 $171 \times 171 = 29241$ $172 \times 172 = 29584$ $173 \times 173 = 29929$ $174 \times 174 = 30276$ $175 \times 175 = 30625$
 $176 \times 176 = 30976$ $177 \times 177 = 31329$ $178 \times 178 = 31684$ $179 \times 179 = 32041$ $180 \times 180 = 32400$
 $181 \times 181 = 32761$ $182 \times 182 = 33124$ $183 \times 183 = 33489$ $184 \times 184 = 33856$ $185 \times 185 = 34225$
 $186 \times 186 = 34596$ $187 \times 187 = 34969$ $188 \times 188 = 35344$ $189 \times 189 = 35721$ $190 \times 190 = 36100$
 $191 \times 191 = 36481$ $192 \times 192 = 36864$ $193 \times 193 = 37249$ $194 \times 194 = 37636$ $195 \times 195 = 38025$
 $196 \times 196 = 38416$ $197 \times 197 = 38809$ $198 \times 198 = 39204$ $199 \times 199 = 39601$ $200 \times 200 = 40000$

1. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

2. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(2)$ and $(g \circ f)(2)$.

3. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-1)$ and $(g \circ f)(-1)$.

4. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(3)$ and $(g \circ f)(3)$.

5. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(0)$ and $(g \circ f)(0)$.

6. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-2)$ and $(g \circ f)(-2)$.

7. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(4)$ and $(g \circ f)(4)$.

8. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(1)$ and $(g \circ f)(1)$.

9. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-3)$ and $(g \circ f)(-3)$.

10. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(5)$ and $(g \circ f)(5)$.

11. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-4)$ and $(g \circ f)(-4)$.

12. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(6)$ and $(g \circ f)(6)$.

13. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-5)$ and $(g \circ f)(-5)$.

14. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(7)$ and $(g \circ f)(7)$.

15. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-6)$ and $(g \circ f)(-6)$.

16. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(8)$ and $(g \circ f)(8)$.

17. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-7)$ and $(g \circ f)(-7)$.

18. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(9)$ and $(g \circ f)(9)$.

19. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(-8)$ and $(g \circ f)(-8)$.

20. Let $f(x) = x^2 + 3x - 7$ and $g(x) = 2x - 5$. Find $(f \circ g)(10)$ and $(g \circ f)(10)$.

1. The function $f(x) = x^2 + 3x - 5$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 2$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 7.

2. The function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 1$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

3. The function $f(x) = x^3 - 6x^2 + 9x - 4$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 2$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

4. The function $f(x) = x^3 - 3x^2 + 2x - 1$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 1$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

5. The function $f(x) = x^3 - 4x^2 + 6x - 3$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 2$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

6. The function $f(x) = x^3 - 5x^2 + 7x - 2$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 1$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

7. The function $f(x) = x^3 - 7x^2 + 10x - 4$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 2$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

8. The function $f(x) = x^3 - 9x^2 + 12x - 5$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 1$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

9. The function $f(x) = x^3 - 6x^2 + 9x - 4$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 2$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 2$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

10. The function $f(x) = x^3 - 3x^2 + 2x - 1$ is defined for $x \in \mathbb{R}$.
 (a) Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 1$.
 (b) Find the equation of the normal to the curve $y = f(x)$ at the point where $x = 1$.
 (c) Find the coordinates of the point on the curve $y = f(x)$ where the gradient of the tangent is 0.

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Braille text representing mathematical content, likely a calculus problem or solution, consisting of multiple lines of Braille characters.

1. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the domain of $f(x)$.

2. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the range of $f(x)$.

3. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the x-intercepts of $f(x)$.

4. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the y-intercept of $f(x)$.

5. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the vertex of $f(x)$.

6. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the maximum value of $f(x)$.

7. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the minimum value of $f(x)$.

8. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the equation of the line of symmetry of $f(x)$.

9. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the equation of the tangent line to the graph of $f(x)$ at the point $(1, 0)$.

10. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the equation of the normal line to the graph of $f(x)$ at the point $(1, 0)$.

11. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the area of the region bounded by the graph of $f(x)$ and the x-axis.

12. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the volume of the solid generated by revolving the graph of $f(x)$ about the y-axis.

13. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the length of the arc of the graph of $f(x)$ from $x = -4$ to $x = 1$.

14. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the average value of $f(x)$ on the interval $[-4, 1]$.

15. The function $f(x) = x^2 + 3x - 4$ is defined for all real numbers. Find the definite integral $\int_{-4}^1 f(x) dx$.

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∫₀¹ x² dx = [$\frac{1}{3}x^3$]₀¹ = $\frac{1}{3}(1)^3 - \frac{1}{3}(0)^3 = \frac{1}{3}$

∫₀¹ x³ dx = [$\frac{1}{4}x^4$]₀¹ = $\frac{1}{4}(1)^4 - \frac{1}{4}(0)^4 = \frac{1}{4}$

∫₀¹ x⁴ dx = [$\frac{1}{5}x^5$]₀¹ = $\frac{1}{5}(1)^5 - \frac{1}{5}(0)^5 = \frac{1}{5}$

∫₀¹ x⁵ dx = [$\frac{1}{6}x^6$]₀¹ = $\frac{1}{6}(1)^6 - \frac{1}{6}(0)^6 = \frac{1}{6}$

∫₀¹ x⁶ dx = [$\frac{1}{7}x^7$]₀¹ = $\frac{1}{7}(1)^7 - \frac{1}{7}(0)^7 = \frac{1}{7}$

∫₀¹ x⁷ dx = [$\frac{1}{8}x^8$]₀¹ = $\frac{1}{8}(1)^8 - \frac{1}{8}(0)^8 = \frac{1}{8}$

∫₀¹ x⁸ dx = [$\frac{1}{9}x^9$]₀¹ = $\frac{1}{9}(1)^9 - \frac{1}{9}(0)^9 = \frac{1}{9}$

∫₀¹ x⁹ dx = [$\frac{1}{10}x^{10}$]₀¹ = $\frac{1}{10}(1)^{10} - \frac{1}{10}(0)^{10} = \frac{1}{10}$

∫₀¹ x¹⁰ dx = [$\frac{1}{11}x^{11}$]₀¹ = $\frac{1}{11}(1)^{11} - \frac{1}{11}(0)^{11} = \frac{1}{11}$

∫₀¹ x¹¹ dx = [$\frac{1}{12}x^{12}$]₀¹ = $\frac{1}{12}(1)^{12} - \frac{1}{12}(0)^{12} = \frac{1}{12}$

∫₀¹ x¹² dx = [$\frac{1}{13}x^{13}$]₀¹ = $\frac{1}{13}(1)^{13} - \frac{1}{13}(0)^{13} = \frac{1}{13}$

∫₀¹ x¹³ dx = [$\frac{1}{14}x^{14}$]₀¹ = $\frac{1}{14}(1)^{14} - \frac{1}{14}(0)^{14} = \frac{1}{14}$

∫₀¹ x¹⁴ dx = [$\frac{1}{15}x^{15}$]₀¹ = $\frac{1}{15}(1)^{15} - \frac{1}{15}(0)^{15} = \frac{1}{15}$

∫₀¹ x¹⁵ dx = [$\frac{1}{16}x^{16}$]₀¹ = $\frac{1}{16}(1)^{16} - \frac{1}{16}(0)^{16} = \frac{1}{16}$

∫₀¹ x¹⁶ dx = [$\frac{1}{17}x^{17}$]₀¹ = $\frac{1}{17}(1)^{17} - \frac{1}{17}(0)^{17} = \frac{1}{17}$

∫₀¹ x¹⁷ dx = [$\frac{1}{18}x^{18}$]₀¹ = $\frac{1}{18}(1)^{18} - \frac{1}{18}(0)^{18} = \frac{1}{18}$

∫₀¹ x¹⁸ dx = [$\frac{1}{19}x^{19}$]₀¹ = $\frac{1}{19}(1)^{19} - \frac{1}{19}(0)^{19} = \frac{1}{19}$

∫₀¹ x¹⁹ dx = [$\frac{1}{20}x^{20}$]₀¹ = $\frac{1}{20}(1)^{20} - \frac{1}{20}(0)^{20} = \frac{1}{20}$

∫₀¹ x²⁰ dx = [$\frac{1}{21}x^{21}$]₀¹ = $\frac{1}{21}(1)^{21} - \frac{1}{21}(0)^{21} = \frac{1}{21}$

∫₀¹ x²¹ dx = [$\frac{1}{22}x^{22}$]₀¹ = $\frac{1}{22}(1)^{22} - \frac{1}{22}(0)^{22} = \frac{1}{22}$

∫₀¹ x²² dx = [$\frac{1}{23}x^{23}$]₀¹ = $\frac{1}{23}(1)^{23} - \frac{1}{23}(0)^{23} = \frac{1}{23}$

∫₀¹ x²³ dx = [$\frac{1}{24}x^{24}$]₀¹ = $\frac{1}{24}(1)^{24} - \frac{1}{24}(0)^{24} = \frac{1}{24}$

∫₀¹ x²⁴ dx = [$\frac{1}{25}x^{25}$]₀¹ = $\frac{1}{25}(1)^{25} - \frac{1}{25}(0)^{25} = \frac{1}{25}$

∫₀¹ x²⁵ dx = [$\frac{1}{26}x^{26}$]₀¹ = $\frac{1}{26}(1)^{26} - \frac{1}{26}(0)^{26} = \frac{1}{26}$

∫₀¹ x²⁶ dx = [$\frac{1}{27}x^{27}$]₀¹ = $\frac{1}{27}(1)^{27} - \frac{1}{27}(0)^{27} = \frac{1}{27}$

∫₀¹ x²⁷ dx = [$\frac{1}{28}x^{28}$]₀¹ = $\frac{1}{28}(1)^{28} - \frac{1}{28}(0)^{28} = \frac{1}{28}$

∫₀¹ x²⁸ dx = [$\frac{1}{29}x^{29}$]₀¹ = $\frac{1}{29}(1)^{29} - \frac{1}{29}(0)^{29} = \frac{1}{29}$

∫₀¹ x²⁹ dx = [$\frac{1}{30}x^{30}$]₀¹ = $\frac{1}{30}(1)^{30} - \frac{1}{30}(0)^{30} = \frac{1}{30}$

∫₀¹ x³⁰ dx = [$\frac{1}{31}x^{31}$]₀¹ = $\frac{1}{31}(1)^{31} - \frac{1}{31}(0)^{31} = \frac{1}{31}$

∫₀¹ x³¹ dx = [$\frac{1}{32}x^{32}$]₀¹ = $\frac{1}{32}(1)^{32} - \frac{1}{32}(0)^{32} = \frac{1}{32}$

∫₀¹ x³² dx = [$\frac{1}{33}x^{33}$]₀¹ = $\frac{1}{33}(1)^{33} - \frac{1}{33}(0)^{33} = \frac{1}{33}$

∫₀¹ x³³ dx = [$\frac{1}{34}x^{34}$]₀¹ = $\frac{1}{34}(1)^{34} - \frac{1}{34}(0)^{34} = \frac{1}{34}$

∫₀¹ x³⁴ dx = [$\frac{1}{35}x^{35}$]₀¹ = $\frac{1}{35}(1)^{35} - \frac{1}{35}(0)^{35} = \frac{1}{35}$

∫₀¹ x³⁵ dx = [$\frac{1}{36}x^{36}$]₀¹ = $\frac{1}{36}(1)^{36} - \frac{1}{36}(0)^{36} = \frac{1}{36}$

∫₀¹ x³⁶ dx = [$\frac{1}{37}x^{37}$]₀¹ = $\frac{1}{37}(1)^{37} - \frac{1}{37}(0)^{37} = \frac{1}{37}$

∫₀¹ x³⁷ dx = [$\frac{1}{38}x^{38}$]₀¹ = $\frac{1}{38}(1)^{38} - \frac{1}{38}(0)^{38} = \frac{1}{38}$

∫₀¹ x³⁸ dx = [$\frac{1}{39}x^{39}$]₀¹ = $\frac{1}{39}(1)^{39} - \frac{1}{39}(0)^{39} = \frac{1}{39}$

∫₀¹ x³⁹ dx = [$\frac{1}{40}x^{40}$]₀¹ = $\frac{1}{40}(1)^{40} - \frac{1}{40}(0)^{40} = \frac{1}{40}$

∫₀¹ x⁴⁰ dx = [$\frac{1}{41}x^{41}$]₀¹ = $\frac{1}{41}(1)^{41} - \frac{1}{41}(0)^{41} = \frac{1}{41}$

∫₀¹ x⁴¹ dx = [$\frac{1}{42}x^{42}$]₀¹ = $\frac{1}{42}(1)^{42} - \frac{1}{42}(0)^{42} = \frac{1}{42}$

∫₀¹ x⁴² dx = [$\frac{1}{43}x^{43}$]₀¹ = $\frac{1}{43}(1)^{43} - \frac{1}{43}(0)^{43} = \frac{1}{43}$

∫₀¹ x⁴³ dx = [$\frac{1}{44}x^{44}$]₀¹ = $\frac{1}{44}(1)^{44} - \frac{1}{44}(0)^{44} = \frac{1}{44}$

∫₀¹ x⁴⁴ dx = [$\frac{1}{45}x^{45}$]₀¹ = $\frac{1}{45}(1)^{45} - \frac{1}{45}(0)^{45} = \frac{1}{45}$

∫₀¹ x⁴⁵ dx = [$\frac{1}{46}x^{46}$]₀¹ = $\frac{1}{46}(1)^{46} - \frac{1}{46}(0)^{46} = \frac{1}{46}$

∫₀¹ x⁴⁶ dx = [$\frac{1}{47}x^{47}$]₀¹ = $\frac{1}{47}(1)^{47} - \frac{1}{47}(0)^{47} = \frac{1}{47}$

∫₀¹ x⁴⁷ dx = [$\frac{1}{48}x^{48}$]₀¹ = $\frac{1}{48}(1)^{48} - \frac{1}{48}(0)^{48} = \frac{1}{48}$

∫₀¹ x⁴⁸ dx = [$\frac{1}{49}x^{49}$]₀¹ = $\frac{1}{49}(1)^{49} - \frac{1}{49}(0)^{49} = \frac{1}{49}$

∫₀¹ x⁴⁹ dx = [$\frac{1}{50}x^{50}$]₀¹ = $\frac{1}{50}(1)^{50} - \frac{1}{50}(0)^{50} = \frac{1}{50}$

∫₀¹ x⁵⁰ dx = [$\frac{1}{51}x^{51}$]₀¹ = $\frac{1}{51}(1)^{51} - \frac{1}{51}(0)^{51} = \frac{1}{51}$

∫₀¹ x⁵¹ dx = [$\frac{1}{52}x^{52}$]₀¹ = $\frac{1}{52}(1)^{52} - \frac{1}{52}(0)^{52} = \frac{1}{52}$

∫₀¹ x⁵² dx = [$\frac{1}{53}x^{53}$]₀¹ = $\frac{1}{53}(1)^{53} - \frac{1}{53}(0)^{53} = \frac{1}{53}$

∫₀¹ x⁵³ dx = [$\frac{1}{54}x^{54}$]₀¹ = $\frac{1}{54}(1)^{54} - \frac{1}{54}(0)^{54} = \frac{1}{54}$

∫₀¹ x⁵⁴ dx = [$\frac{1}{55}x^{55}$]₀¹ = $\frac{1}{55}(1)^{55} - \frac{1}{55}(0)^{55} = \frac{1}{55}$

∫₀¹ x⁵⁵ dx = [$\frac{1}{56}x^{56}$]₀¹ = $\frac{1}{56}(1)^{56} - \frac{1}{56}(0)^{56} = \frac{1}{56}$

∫₀¹ x⁵⁶ dx = [$\frac{1}{57}x^{57}$]₀¹ = $\frac{1}{57}(1)^{57} - \frac{1}{57}(0)^{57} = \frac{1}{57}$

∫₀¹ x⁵⁷ dx = [$\frac{1}{58}x^{58}$]₀¹ = $\frac{1}{58}(1)^{58} - \frac{1}{58}(0)^{58} = \frac{1}{58}$

∫₀¹ x⁵⁸ dx = [$\frac{1}{59}x^{59}$]₀¹ = $\frac{1}{59}(1)^{59} - \frac{1}{59}(0)^{59} = \frac{1}{59}$

∫₀¹ x⁵⁹ dx = [$\frac{1}{60}x^{60}$]₀¹ = $\frac{1}{60}(1)^{60} - \frac{1}{60}(0)^{60} = \frac{1}{60}$

∫₀¹ x⁶⁰ dx = [$\frac{1}{61}x^{61}$]₀¹ = $\frac{1}{61}(1)^{61} - \frac{1}{61}(0)^{61} = \frac{1}{61}$

∫₀¹ x⁶¹ dx = [$\frac{1}{62}x^{62}$]₀¹ = $\frac{1}{62}(1)^{62} - \frac{1}{62}(0)^{62} = \frac{1}{62}$

∫₀¹ x⁶² dx = [$\frac{1}{63}x^{63}$]₀¹ = $\frac{1}{63}(1)^{63} - \frac{1}{63}(0)^{63} = \frac{1}{63}$

∫₀¹ x⁶³ dx = [$\frac{1}{64}x^{64}$]₀¹ = $\frac{1}{64}(1)^{64} - \frac{1}{64}(0)^{64} = \frac{1}{64}$

∫₀¹ x⁶⁴ dx = [$\frac{1}{65}x^{65}$]₀¹ = $\frac{1}{65}(1)^{65} - \frac{1}{65}(0)^{65} = \frac{1}{65}$

∫₀¹ x⁶⁵ dx = [$\frac{1}{66}x^{66}$]₀¹ = $\frac{1}{66}(1)^{66} - \frac{1}{66}(0)^{66} = \frac{1}{66}$

∫₀¹ x⁶⁶ dx = [$\frac{1}{67}x^{67}$]₀¹ = $\frac{1}{67}(1)^{67} - \frac{1}{67}(0)^{67} = \frac{1}{67}$

∫₀¹ x⁶⁷ dx = [$\frac{1}{68}x^{68}$]₀¹ = $\frac{1}{68}(1)^{68} - \frac{1}{68}(0)^{68} = \frac{1}{68}$

∫₀¹ x⁶⁸ dx = [$\frac{1}{69}x^{69}$]₀¹ = $\frac{1}{69}(1)^{69} - \frac{1}{69}(0)^{69} = \frac{1}{69}$

∫₀¹ x⁶⁹ dx = [$\frac{1}{70}x^{70}$]₀¹ = $\frac{1}{70}(1)^{70} - \frac{1}{70}(0)^{70} = \frac{1}{70}$

∫₀¹ x⁷⁰ dx = [$\frac{1}{71}x^{71}$]₀¹ = $\frac{1}{71}(1)^{71} - \frac{1}{71}(0)^{71} = \frac{1}{71}$

∫₀¹ x⁷¹ dx = [$\frac{1}{72}x^{72}$]₀¹ = $\frac{1}{72}(1)^{72} - \frac{1}{72}(0)^{72} = \frac{1}{72}$

∫₀¹ x⁷² dx = [$\frac{1}{73}x^{73}$]₀¹ = $\frac{1}{73}(1)^{73} - \frac{1}{73}(0)^{73} = \frac{1}{73}$

∫₀¹ x⁷³ dx = [$\frac{1}{74}x^{74}$]₀¹ = $\frac{1}{74}(1)^{74} - \frac{1}{74}(0)^{74} = \frac{1}{74}$

∫₀¹ x⁷⁴ dx = [$\frac{1}{75}x^{75}$]₀¹ = $\frac{1}{75}(1)^{75} - \frac{1}{75}(0)^{75} = \frac{1}{75}$

∫₀¹ x⁷⁵ dx = [$\frac{1}{76}x^{76}$]₀¹ = $\frac{1}{76}(1)^{76} - \frac{1}{76}(0)^{76} = \frac{1}{76}$

∫₀¹ x⁷⁶ dx = [$\frac{1}{77}x^{77}$]₀¹ = $\frac{1}{77}(1)^{77} - \frac{1}{77}(0)^{77} = \frac{1}{77}$

∫₀¹ x⁷⁷ dx = [$\frac{1}{78}x^{78}$]₀¹ = $\frac{1}{78}(1)^{78} - \frac{1}{78}(0)^{78} = \frac{1}{78}$

∫₀¹ x⁷⁸ dx = [$\frac{1}{79}x^{79}$]₀¹ = $\frac{1}{79}(1)^{79} - \frac{1}{79}(0)^{79} = \frac{1}{79}$

∫₀¹ x⁷⁹ dx = [$\frac{1}{80}x^{80}$]₀¹ = $\frac{1}{80}(1)^{80} - \frac{1}{80}(0)^{80} = \frac{1}{80}$

∫₀¹ x⁸⁰ dx = [$\frac{1}{81}x^{81}$]₀¹ = $\frac{1}{81}(1)^{81} - \frac{1}{81}(0)^{81} = \frac{1}{81}$

∫₀¹ x⁸¹ dx = [$\frac{1}{82}x^{82}$]₀¹ = $\frac{1}{82}(1)^{82} - \frac{1}{82}(0)^{82} = \frac{1}{82}$

∫₀¹ x⁸² dx = [$\frac{1}{83}x^{83}$]₀¹ = $\frac{1}{83}(1)^{83} - \frac{1}{83}(0)^{83} = \frac{1}{83}$

∫₀¹ x⁸³ dx = [$\frac{1}{84}x^{84}$]₀¹ = $\frac{1}{84}(1)^{84} - \frac{1}{84}(0)^{84} = \frac{1}{84}$

∫₀¹ x⁸⁴ dx = [$\frac{1}{85}x^{85}$]₀¹ = $\frac{1}{85}(1)^{85} - \frac{1}{85}(0)^{85} = \frac{1}{85}$

∫₀¹ x⁸⁵ dx = [$\frac{1}{86}x^{86}$]₀¹ = $\frac{1}{86}(1)^{86} - \frac{1}{86}(0)^{86} = \frac{1}{86}$

∫₀¹ x⁸⁶ dx = [$\frac{1}{87}x^{87}$]₀¹ = $\frac{1}{87}(1)^{87} - \frac{1}{87}(0)^{87} = \frac{1}{87}$

∫₀¹ x⁸⁷ dx = [$\frac{1}{88}x^{88}$]₀¹ = $\frac{1}{88}(1)^{88} - \frac{1}{88}(0)^{88} = \frac{1}{88}$

∫₀¹ x⁸⁸ dx = [$\frac{1}{89}x^{89}$]₀¹ = $\frac{1}{89}(1)^{89} - \frac{1}{89}(0)^{89} = \frac{1}{89}$

∫₀¹ x⁸⁹ dx = [$\frac{1}{90}x^{90}$]₀¹ = $\frac{1}{90}(1)^{90} - \frac{1}{90}(0)^{90} = \frac{1}{90}$

∫₀¹ x⁹⁰ dx = [$\frac{1}{91}x^{91}$]₀¹ = $\frac{1}{91}(1)^{91} - \frac{1}{91}(0)^{91} = \frac{1}{91}$

∫₀¹ x⁹¹ dx = [$\frac{1}{92}x^{92}$]₀¹ = $\frac{1}{92}(1)^{92} - \frac{1}{92}(0)^{92} = \frac{1}{92}$

∫₀¹ x⁹² dx = [$\frac{1}{93}x^{93}$]₀¹ = $\frac{1}{93}(1)^{93} - \frac{1}{93}(0)^{93} = \frac{1}{93}$

∫₀¹ x⁹³ dx = [$\frac{1}{94}x^{94}$]₀¹ = $\frac{1}{94}(1)^{94} - \frac{1}{94}(0)^{94} = \frac{1}{94}$

∫₀¹ x⁹⁴ dx = [$\frac{1}{95}x^{95}$]₀¹ = $\frac{1}{95}(1)^{95} - \frac{1}{95}(0)^{95} = \frac{1}{95}$

∫₀¹ x⁹⁵ dx = [$\frac{1}{96}x^{96}$]₀¹ = $\frac{1}{96}(1)^{96} - \frac{1}{96}(0)^{96} = \frac{1}{96}$

∫₀¹ x⁹⁶ dx = [$\frac{1}{97}x^{97}$]₀¹ = $\frac{1}{97}(1)^{97} - \frac{1}{97}(0)^{97} = \frac{1}{97}$

∫₀¹ x⁹⁷ dx = [$\frac{1}{98}x^{98}$]₀¹ = $\frac{1}{98}(1)^{98} - \frac{1}{98}(0)^{98} = \frac{1}{98}$

∫₀¹ x⁹⁸ dx = [$\frac{1}{99}x^{99}$]₀¹ = $\frac{1}{99}(1)^{99} - \frac{1}{99}(0)^{99} = \frac{1}{99}$

∫₀¹ x⁹⁹ dx = [$\frac{1}{100}x^{100}$]₀¹ = $\frac{1}{100}(1)^{100} - \frac{1}{100}(0)^{100} = \frac{1}{100}$

The derivative of $\tan x$ is $\sec^2 x$. To find the derivative of $\tan x$, we use the quotient rule. We write $\tan x = \frac{\sin x}{\cos x}$. Then, the derivative is $\frac{\cos x \cdot \cos x - \sin x \cdot (-\sin x)}{\cos^2 x} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$.

The derivative of $\cot x$ is $-\csc^2 x$. To find the derivative of $\cot x$, we use the quotient rule. We write $\cot x = \frac{\cos x}{\sin x}$. Then, the derivative is $\frac{\sin x \cdot (-\sin x) - \cos x \cdot \cos x}{\sin^2 x} = \frac{-\sin^2 x - \cos^2 x}{\sin^2 x} = \frac{-(\sin^2 x + \cos^2 x)}{\sin^2 x} = \frac{-1}{\sin^2 x} = -\csc^2 x$.

The derivative of $\csc x$ is $-\csc x \cot x$. To find the derivative of $\csc x$, we use the quotient rule. We write $\csc x = \frac{1}{\sin x}$. Then, the derivative is $\frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \csc x$.

The derivative of $\sec x$ is $\sec x \tan x$. To find the derivative of $\sec x$, we use the quotient rule. We write $\sec x = \frac{1}{\cos x}$. Then, the derivative is $\frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$.

The derivative of $\csc x$ is $-\csc x \cot x$. To find the derivative of $\csc x$, we use the quotient rule. We write $\csc x = \frac{1}{\sin x}$. Then, the derivative is $\frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \csc x$.

The derivative of $\sec x$ is $\sec x \tan x$. To find the derivative of $\sec x$, we use the quotient rule. We write $\sec x = \frac{1}{\cos x}$. Then, the derivative is $\frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$.

The derivative of $\csc x$ is $-\csc x \cot x$. To find the derivative of $\csc x$, we use the quotient rule. We write $\csc x = \frac{1}{\sin x}$. Then, the derivative is $\frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \csc x$.

The derivative of $\sec x$ is $\sec x \tan x$. To find the derivative of $\sec x$, we use the quotient rule. We write $\sec x = \frac{1}{\cos x}$. Then, the derivative is $\frac{\cos x \cdot 0 - 1 \cdot (-\sin x)}{\cos^2 x} = \frac{\sin x}{\cos^2 x} = \frac{\sin x}{\cos x} \cdot \frac{1}{\cos x} = \tan x \sec x$.

The derivative of $\csc x$ is $-\csc x \cot x$. To find the derivative of $\csc x$, we use the quotient rule. We write $\csc x = \frac{1}{\sin x}$. Then, the derivative is $\frac{\sin x \cdot 0 - 1 \cdot \cos x}{\sin^2 x} = \frac{-\cos x}{\sin^2 x} = -\frac{\cos x}{\sin x} \cdot \frac{1}{\sin x} = -\cot x \csc x$.

The function $f(x) = x^2 - 3x + 2$ is a parabola opening upwards. The vertex is at $(1.5, -2.25)$. The x-intercepts are at $x = 1$ and $x = 2$. The y-intercept is at $(0, 2)$. The function is positive for $x < 1$ and $x > 2$, and negative for $1 < x < 2$. The domain is all real numbers, and the range is $y \geq -2.25$.

The function $f(x) = \frac{1}{x}$ is a hyperbola with two branches. One branch is in the first quadrant, and the other is in the third quadrant. The x-axis ($y = 0$) and y-axis ($x = 0$) are asymptotes. The function is increasing on each branch. The domain is $x \neq 0$, and the range is $y \neq 0$.

The function $f(x) = \ln(x)$ is a logarithmic function. It is defined for $x > 0$. The x-axis ($y = 0$) is a horizontal asymptote as $x \rightarrow 0^+$. The function passes through the point $(1, 0)$. It is an increasing function. The domain is $x > 0$, and the range is all real numbers.

The function $f(x) = e^x$ is an exponential function. It is defined for all real numbers x . The x-axis ($y = 0$) is a horizontal asymptote as $x \rightarrow -\infty$. The function passes through the point $(0, 1)$. It is an increasing function. The domain is all real numbers, and the range is $y > 0$.

The function $f(x) = \sin(x)$ is a periodic function with a period of 2π . It oscillates between $y = -1$ and $y = 1$. The function passes through the origin $(0, 0)$. The domain is all real numbers, and the range is $-1 \leq y \leq 1$.

The function $f(x) = \cos(x)$ is a periodic function with a period of 2π . It oscillates between $y = -1$ and $y = 1$. The function passes through the point $(0, 1)$. The domain is all real numbers, and the range is $-1 \leq y \leq 1$.

The function $f(x) = \tan(x)$ is a periodic function with a period of π . It has vertical asymptotes at $x = \frac{\pi}{2} + k\pi$ for any integer k . The function passes through the origin $(0, 0)$. The domain is $x \neq \frac{\pi}{2} + k\pi$, and the range is all real numbers.

The function $f(x) = \cot(x)$ is a periodic function with a period of π . It has vertical asymptotes at $x = k\pi$ for any integer k . The function passes through the point $(\frac{\pi}{2}, 0)$. The domain is $x \neq k\pi$, and the range is all real numbers.

The function $f(x) = \csc(x)$ is a periodic function with a period of 2π . It has vertical asymptotes at $x = k\pi$ for any integer k . The function passes through the point $(\frac{\pi}{2}, 1)$. The domain is $x \neq k\pi$, and the range is $y < -1$ or $y > 1$.

The function $f(x) = \sec(x)$ is a periodic function with a period of 2π . It has vertical asymptotes at $x = \frac{\pi}{2} + k\pi$ for any integer k . The function passes through the point $(0, 1)$. The domain is $x \neq \frac{\pi}{2} + k\pi$, and the range is $y < -1$ or $y > 1$.

The function $f(x) = \arcsin(x)$ is an inverse trigonometric function. It is defined for $-1 \leq x \leq 1$. The range is $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$. The function passes through the origin $(0, 0)$. The domain is $-1 \leq x \leq 1$, and the range is $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$.

The function $f(x) = \arccos(x)$ is an inverse trigonometric function. It is defined for $-1 \leq x \leq 1$. The range is $0 \leq y \leq \pi$. The function passes through the point $(1, 0)$. The domain is $-1 \leq x \leq 1$, and the range is $0 \leq y \leq \pi$.

The function $f(x) = \arctan(x)$ is an inverse trigonometric function. It is defined for all real numbers x . The range is $-\frac{\pi}{2} < y < \frac{\pi}{2}$. The function passes through the origin $(0, 0)$. The domain is all real numbers, and the range is $-\frac{\pi}{2} < y < \frac{\pi}{2}$.

The function $f(x) = \operatorname{arccot}(x)$ is an inverse trigonometric function. It is defined for all real numbers x . The range is $0 < y < \pi$. The function passes through the point $(1, \frac{\pi}{2})$. The domain is all real numbers, and the range is $0 < y < \pi$.

∫₀¹ (x² + 3x + 2) dx = [$\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x$]₀¹ = $\frac{1}{3} + \frac{3}{2} + 2 = \frac{17}{6}$

∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

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∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

∫₀¹ (x² + 3x + 2) dx = $\frac{17}{6}$

1. The function $f(x) = 3x^2 - 5x + 2$ is defined for $x \in \mathbb{R}$.
Find the gradient of the tangent to the curve $y = f(x)$ at the point where $x = 4$.
Hence find the equation of the tangent to the curve at this point.

2. The function $f(x) = 2x^3 - 9x^2 + 12x - 5$ is defined for $x \in \mathbb{R}$.
Find the values of x for which $f(x)$ is a local maximum or minimum.
Hence find the nature of each stationary point.

3. The function $f(x) = x^3 - 3x^2 + 2x + 1$ is defined for $x \in \mathbb{R}$.
Find the values of x for which $f(x)$ is a local maximum or minimum.
Hence find the nature of each stationary point.

4. The function $f(x) = x^3 - 3x^2 + 2x + 1$ is defined for $x \in \mathbb{R}$.
Find the values of x for which $f(x)$ is a local maximum or minimum.
Hence find the nature of each stationary point.

5. The function $f(x) = x^3 - 3x^2 + 2x + 1$ is defined for $x \in \mathbb{R}$.
Find the values of x for which $f(x)$ is a local maximum or minimum.
Hence find the nature of each stationary point.



Sample 4. Teaching Material

This sample is transcribed using *The Nemeth Braille Code for Mathematics and Science Notation 1972 Revision* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

Teaching Material in Nemeth Code

Nemeth Symbols

⠠	punctuation indicator	⠠⠠	begin fraction portion of mixed number
⠠⠠	general omission symbol	⠠⠠⠠	end fraction portion of mixed number
⠠⠠⠠	begin modified expression	⠠⠠⠠⠠	begin complex fraction
⠠⠠⠠⠠	begin modifier above	⠠⠠⠠⠠⠠	end complex fraction
⠠⠠⠠⠠⠠	end modified expression	⠠⠠⠠⠠⠠⠠	horizontal complex fraction line
⠠⠠⠠⠠⠠⠠	bold next letter	⠠⠠⠠	opening parenthesis
⠠⠠⠠⠠	plus sign	⠠⠠⠠	closing parenthesis
⠠⠠⠠	minus sign	⠠⠠⠠⠠	opening bracket
⠠⠠⠠⠠	times sign (cross)	⠠⠠⠠⠠	closing bracket
⠠⠠⠠	times sign (dot)	⠠⠠⠠⠠⠠	opening brace
⠠⠠⠠⠠⠠	divided by sign	⠠⠠⠠⠠⠠	closing brace
⠠⠠⠠	divided by sign (spatial)	⠠⠠⠠	vertical bar
⠠⠠⠠⠠	plus or minus sign	⠠⠠⠠⠠⠠	enlarged opening bracket
⠠⠠⠠⠠	equal sign	⠠⠠⠠⠠⠠	enlarged closing bracket
⠠⠠⠠⠠⠠	not equal sign	⠠⠠⠠	decimal point
⠠⠠⠠⠠⠠⠠	equal sign under tilde	⠠⠠⠠⠠	percent sign
⠠⠠⠠⠠	less than sign	⠠⠠	mathematical comma
⠠⠠⠠⠠	greater than sign	⠠⠠⠠	dollar sign
⠠⠠⠠⠠⠠	bar under less than	⠠⠠	factorial sign
⠠⠠⠠⠠⠠	bar under greater than	⠠⠠	begin radical
⠠⠠	begin fraction	⠠⠠	end radical
⠠⠠	end fraction	⠠⠠	index of radical indicator
⠠⠠	horizontal fraction line		
⠠⠠⠠	diagonal fraction line		

Continued on next left-hand page.

Teaching Material in UEBC

UEBC Symbols

⠠	grade one symbol	⠠⠠	closing parenthesis
⠠⠠	grade one word	⠠⠠⠠	vertical line
⠠⠠⠠	begin grade one passage	⠠⠠⠠	opening brace
⠠	(before a space)	⠠⠠⠠	closing brace
⠠⠠⠠	end grade one passage	⠠⠠⠠	opening bracket
⠠⠠⠠	plus sign	⠠⠠⠠	closing bracket
⠠⠠⠠	minus sign	⠠⠠⠠⠠	multi-line opening bracket
⠠⠠⠠	times sign (cross)	⠠⠠⠠⠠	multi-line closing bracket
⠠⠠⠠	times sign (dot)	⠠⠠	begin compound item
⠠⠠⠠	divided by sign	⠠⠠	end compound item
⠠⠠⠠	divided by sign (spatial)	⠠⠠	decimal point
⠠⠠⠠	plus or minus sign	⠠⠠⠠	percent sign
⠠⠠⠠	equal sign	⠠⠠⠠	dollar sign
⠠⠠⠠	less than sign	⠠⠠	factorial sign
⠠⠠⠠	greater than sign	⠠⠠⠠	line through previous item
⠠⠠⠠⠠	less than or equal sign	⠠⠠	begin radical
⠠⠠⠠⠠	greater than or equal sign	⠠⠠⠠	begin radical with next item as index of radical
⠠⠠⠠⠠	tilde over equal sign	⠠⠠	end radical
⠠⠠⠠	tilde	⠠⠠	superscript next item
⠠⠠⠠⠠	much less than sign	⠠⠠	subscript next item
⠠⠠	numeric fraction line	⠠⠠⠠	next item directly above previous item
⠠⠠	begin fraction	⠠⠠⠠	caret over previous item
⠠⠠	end fraction	⠠⠠	bar over previous item
⠠⠠⠠	fraction line	⠠⠠⠠	right arrow over previous item
⠠⠠⠠	slash		
⠠⠠⠠	opening parenthesis		

Continued on next right-hand page.

Nemeth Symbols (cont.)

⠠	begin superscript	⠠	horizontal bar
⠡	begin subscript	⠠⠠	parallel sign
⠢	comma within subscript	⠠⠠	perpendicular sign
⠣	return to baseline after superscript or subscript	⠠⠠⠠	degree sign
⠠⠠	crosshatch	⠠⠠	ratio sign
⠠⠠	angle sign	⠠⠠⠠⠠	double dash
⠠⠠	caret	⠠⠠	tilde
⠠⠠	arc	⠠⠠	infinity sign
⠠⠠	triangle	⠠⠠⠠⠠⠠⠠	two nested less than signs
⠠⠠	square	⠠⠠⠠	Greek capital delta
⠠⠠	circle	⠠⠠	Greek gamma
⠠⠠⠠⠠⠠	right and left barbed arrow	⠠⠠⠠	Greek capital sigma
⠠⠠	right arrow		

UEBC Symbols (cont.)

⠠⠠	crosshatch, or "number"	⠠⠠⠠⠠	blank (A UEBC symbol has not yet been assigned.)
⠠⠠⠠	transcriber-defined shape: angle	⠠⠠	bold word
⠠⠠⠠	transcriber-defined shape: arc	⠠⠠	bold symbol
⠠⠠⠠	triangle	⠠⠠⠠	begin capitalized passage
⠠⠠⠠	square	⠠⠠	end capitals within a sequence
⠠⠠	circle	⠠	(before a space) end capitalized passage
⠠⠠⠠	transcriber-defined shape: parallel sign	⠠⠠	begin italic passage
⠠⠠⠠	transcriber-defined shape: perpendicular sign	⠠	(before a space) end italic passage
⠠	end shape	⠠⠠	infinity sign
⠠⠠	right arrow	⠠⠠⠠	ellipsis
⠠⠠⠠⠠	left- and right-pointing arrow	⠠⠠⠠	capital Greek delta
⠠⠠	degree sign	⠠⠠	Greek gamma
⠠	(spaced) ratio sign	⠠⠠⠠	capital Greek sigma



Sample 4a. Math Workshop

A. Arithmetic

1. addition and subtraction

$$4 + 5 =$$

$$15 - 8 =$$

2. multiplication

$$29 \times 6 =$$

$$453 \cdot 80 =$$

3. division

$$55 \div 5 = 11$$

$$24 \overline{)9785}$$

4. fractions

$$\frac{1}{2} \cdot \frac{1}{4} =$$

$$\frac{5}{45} + \frac{6}{9} + \frac{7}{18} =$$

5. mixed numbers

$$23\frac{1}{4} \times 2 =$$

$$2\frac{1}{2} + 1\frac{3}{4} =$$

6. decimals

$$45.6 + 2.4 =$$

$$978.2 \div 12 =$$

7. word problems (% , \$)

A suit that sold for \$120 is on sale for \$75. The percent of decrease is 37.5%.

8. comparison symbols <, >, =

$$5 = 8 \text{ is false}$$

B. Algebra

1. factorial and absolute value

$$5! = 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1$$

$$5! = 120$$

$$|-34.7|$$

2. negative numbers

$$-4.5 - 16.3 =$$

$$-12.2 + 3.7 =$$

3. variables, linear equations, grouping symbols

$$y = 3x + 5$$

Simplify:

$$\{2x - 4[3x - 5(2x + 3) - 4x] + 6\}$$

4. comparison symbols \leq , \geq , \neq , and the plus or minus sign \pm

$$y \geq 2x - 3$$

5. square roots and radicals with higher indices

$$\sqrt{237}$$

$$\sqrt[3]{99}$$

6. monomials, polynomials and quadratic equations

$$-4z^2$$

$$(3x^2 + 2x - 1)(x^2 - 3)$$

$$(x + 2)^2 =$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

If $3x^2 + 5x - 2 = 0$, then

$$x = \frac{-5 \pm \sqrt{(5)^2 - 4(3)(-2)}}{2(3)}$$

7. rational expressions and complex fractions

$$\frac{4x - x^2}{2x + 3} \div \frac{16x^2 - 4x^3}{8x + 12}$$

$$\frac{\frac{1}{4} + \frac{2}{3}}{\frac{1}{2}} =$$

8. value table and matrix

2	4	6	8	10	12	x
8	16	24	32	40	?	?

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} -2 & 3 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 2 & 19 \\ 1 & 26 \\ 0 & 33 \end{bmatrix}$$

9. table

Grading Period	# of A's
1	5
2	10
3	11

C. Geometry

1. angle and arc

$\angle ABC$

\widehat{EF} \widehat{EF}

2. shapes

$\triangle ABC$

$\square ABCD$

$\circ O$

3. line, line segment and ray

\overleftrightarrow{XY} \overline{AF} \overrightarrow{AB}

4. comparison symbols: \parallel , \perp , \cong

$\overline{AB} \parallel \overline{CD}$

$\overline{AB} \perp \overline{CD}$

$\angle ABC \cong \angle CDE$

5. degrees

$90^\circ + 90^\circ$

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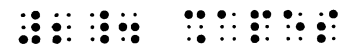
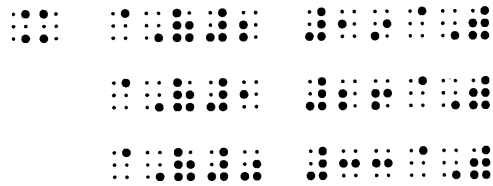
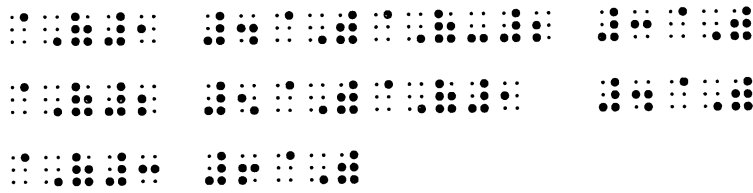
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Sample 4b. Chapter 6 Review

Write each ratio in simplest form.

1. $\frac{AB}{CD}$

2. $\frac{m\angle BAC}{m\angle DAC}$

3. $\frac{BC}{AC}$

4. Determine whether the ratios $\frac{3}{5}$ and $\frac{15}{25}$ form a proportion.

Find the value of x in each proportion.

5. $14 : x = 21 : 27$

6. $\frac{5}{2} = \frac{15}{x}$

7. Complete the statement in five different ways.

If $\frac{2}{3} = \frac{8}{12}$, then _____

8. Use the proportion $\frac{7}{8} = \frac{x}{y}$ to complete the statement.

$\frac{x}{7} = \underline{\hspace{2cm}}$

9. Write a two-column proof.

Given: $\frac{JN}{NM} = \frac{JK}{KL}$

Prove: $\frac{JM}{NM} = \frac{JL}{KL}$

10. $\overline{RS} \parallel \overline{UV}$ Find x and y .

11. Prove $\triangle EFG \sim \triangle EGH$.

12. Find the value of x for which the triangles are similar.

13. Given $\triangle TRS$ and $\triangle VWX$ with $\angle R \cong \angle W$, state the proportion that must be true if $\triangle TRS \sim \triangle VWX$ by SAS Similarity.

Given $\triangle CAB$ and $a \parallel b \parallel c$, complete each statement and state the theorem applied.

14. If $CE = 3$, $CA = 9$, and $CB = 12$, then $CG = \underline{\hspace{2cm}}$

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Sample 4c. Algebra I Chapter 7 Test

1. Match the equation or inequality with the graph it describes.
 - a. $|x| = 1$
 - b. $|x| \leq 1$
 - c. $|x| \geq 1$
2. Which equation matches the graph?
 - (a) $y = \left| \frac{1}{2}x \right| - 3$
 - (b) $y = \left| \frac{1}{2}x + 1 \right|$
 - (c) $y = \left| \frac{1}{2}x \right| + 1$
 - (d) $y = |x| + 1$
3. Solve the equation.
 $|x - 7| = 3$
4. Evaluate.
 $\sqrt{36}$
5. Complete the table. Then graph the equation
 $y = \sqrt{x} - 4$.

x	\sqrt{x}	$\sqrt{x} - 4$	(x, y)
0			
1			
4			
9			
16			
6. Find the taxi distance between $(-3, 1)$ and $(2, -4)$.
7. Use the Distance Formula to find the distance between $(3, 5)$ and $(-2, -3)$.
8. Use the proportion to find the value of x .

$$\frac{x}{63} = \frac{5}{28}$$
9. The two triangles are similar. Find x , y , and z . (Hint: Begin by using the Pythagorean Theorem.)
10. For the triangle shown, find $\sin K$, $\cos K$, and $\tan K$. Give exact answers.
11. A 17-ft ladder makes an angle of 65° with the ground. Give AC , the distance of the bottom of the ladder from the wall. Round your answer to the nearest tenth of a foot.



Sample 4d. Algebra II Chapter 3 Test

Solve each system of linear equations if possible. State the number of solutions, and tell whether the system is consistent or inconsistent.

- $5x + 2y = -4$
 $5x - 2y = -6$
- $y = 2x + 5$
 $y = -7 + 2x$
- $2x + y = 4$
 $2y = 8 - 4x$
- A system of consistent linear equations in two variables has _____ solution(s).
 - 0
 - 1
 - 2
 - at least 1
- A system of inconsistent linear equations in two variables has _____ solution(s).
 - 0
 - 1
 - 2
 - at least 1
- Graph the system of linear inequalities and indicate the solution by an approved method.
 $y \geq -3x - 1$
 $y < \frac{1}{2}x + 3$
- In a linear programming problem, an inequality that models a limitation is called a(n)
 - solution
 - objective function
 - constraint
 - feasible region
- The _____ contains the possible solutions to a linear programming problem.
 - optimal solution
 - feasible region
 - constraint



Sample 4e. A Sample Article

Abstract. This article illustrates many features of a mathematics document created with *Scientific Notebook*.

Mathematics and Text

Let H be a Hilbert space, C be a closed bounded convex subset of H , T a nonexpansive self map of C . Suppose that as $n \rightarrow \infty$, $a_{n,k} \rightarrow 0$ for each k , and $\gamma_n = \sum_{k=0}^{\infty} (a_{n,k+1} - a_{n,k})^+ \rightarrow 0$. Then for each x in C , $A_n x = \sum_{k=0}^{\infty} a_{n,k} T^k x$ converges weakly to a fixed point of T .

In-line and Displayed Mathematics

The equation

$$u_{tt} - \Delta u + u^5 + u|u|^{p-2} = 0 \text{ in } \mathbf{R}^3 \times [0, \infty[\quad 1$$

is numbered and it also would have the label “wave”. You can use this label to jump to this equation using hypertext links.

Multi-Line Displays

Scientific Notebook provides a range of alignment options for multiline mathematical displays. Here is a series of multiline displays.

$$x = 17y \quad 2$$

$$y > \begin{array}{l} a + b + c + d + e + f + g + h + i + j + \\ k + l + m + n + o + p \end{array} \quad 3$$

$$x \ll \begin{array}{l} y_1 + \dots + y_n \\ \leq z \end{array}$$

If $f(x) = x + 1$, then we will have

$$f([x + 1]/[x + 2]) = \{ [x + 1]/[x + 2] \} + 1 = (2x + 3)/(x + 2)$$

$$\begin{aligned} &1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 + 10 + 11 + 12 \\ &\quad + 13 + 14 + 15 + 16 + 17 + 18 + 19 + 20 = 190 \end{aligned}$$

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Sample 5. Computer

This sample is transcribed using *Code for Computer Braille Notation 1987* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

Comment lines in programs begin with apostrophes. Those which are printed at the end of a line of code are brailled on the following line in cell 5. Those which are printed at the beginning of a line are brailled as printed.

A right-pointing arrow is used as a print line-continuation indicator to show that a line of code continues from one line to the next. This is omitted in braille.

Items in *italic* indicate placeholders for information supplied by the reader.

Items inside square brackets are optional.

Sans serif font is used in print for presenting sample code.



Tip Using an update query to change job titles is more efficient.

For more information about bookmarks, search Help for "Bookmark."

Access Basic sets the **NoMatch** property to **True** whenever a **Find** operation fails and the current record position is undefined. There may be a current record, but you have no way to tell which record it may be. If you want to be able to return to the previous current record following a failed **Find** operation, use a bookmark.

NoMatch is **False** whenever the operation succeeds. In this case, the current record position is the record found by one of the **Find** methods.

The following example illustrates how you can use the **FindNext** method to find all records that contain **Null** in the **Phone** field in a table named **Customers**.

```
Dim MyDb As Database, MySet As Recordset, Criterion As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Customers",DB_OPEN_DYNASET)

Criterion = "[Phone] Is Null"

MySet.FindFirst Criterion
Do Until MySet.NoMatch
    MsgBox MySet![Company Name] & " is missing a phone number."
    MySet.FindNext Criterion
Loop
```

Tip You can modify this example to include a call to the **InputBox** function instead of a call to the **MsgBox** function. This lets you enter a phone number for the found records. Then, you can use the **Edit** and **Update** methods to add the new phone numbers to the records.

► Using the Seek Method

For more information about setting the current index, search Help for "current index" or see "Sorting a Recordset" earlier in this chapter.

You use the **Seek** method to locate a record in a table-type **Recordset**. To locate a record in a dynaset or snapshot, use one of the **Find** methods described in the preceding section. When you use **Seek** to locate a record, Access Basic uses the table's current index, as defined by the **Index** property.

The syntax for the **Seek** method is:

```
table.Seek comparison, key1, key2 ...
```

The *table* argument is a recordset variable that refers to the table you are searching through. The **Seek** method accepts a number of arguments, the first of which is *comparison*, a string that determines the kind of comparison that's performed. The following table lists the comparison strings you can use with **Seek**.

Comparison string	Description
"="	Equal to the specified key values
">="	Greater than or equal to the specified key values
Greater than the specified key values	
"<="	Less than or equal to the specified key values
"<"	Less than the specified key values

The *keyn* arguments are a series of one or more values that Access Basic compares to values in the table records. The following example opens a table named Products, and uses the **Seek** method to locate the first record containing a value of 1 in the Supplier ID field (which is a nonunique index field). It changes 1 to 2 and saves the change with the **Update** method. Subsequent passes through the loop locate the next record that satisfies the condition.

```
Dim MyDb As Database, MyTable As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
' Open Table.
Set MyTable = MyDb.OpenRecordset("Products", DB_OPEN_TABLE)

MyTable.Index = "Supplier ID"           ' Define current index.
MyTable.Seek "=", 1                    ' Seek record.

Do Until MyTable.NoMatch                ' Until no record is found.
    MyTable.Edit                         ' Enable editing.
    MyTable("Supplier ID") = 2          ' Change Supplier ID.
    MyTable.Update                       ' Save changes.
    MyTable.Seek "=", 1                  ' Seek next record.
Loop                                     ' End of loop.

MyTable.Close                           ' Close Table.
```

If you use the **Seek** method on a **Recordset** object of type table without first setting the current index, a run-time error occurs.

The next example illustrates how you can create a function that uses the **Seek** method to locate a record using a multiple-field index.

```
Function GetFirstPrice(ByVal OrderID As Long, ByVal ProductID As Long)
    As Variant
```

```
Dim MyDb As Database, MyTable As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyTable = MyDb.OpenRecordset("Order Details", DB_OPEN_TABLE)

MyTable.Index = "PrimaryKey"
MyTable.Seek "=", OrderID, ProductID

If MyTable.NoMatch Then
    GetFirstPrice = Null
    MsgBox "Couldn't find order detail record."
Else
    GetFirstPrice = MyTable![Unit Price]
End If

MyTable.Close
MyDb.Close

End Function
```

For more information about using the **Seek** method, search Help for "Seek."

In this example, the table's primary key consists of two fields, Order ID and Product ID. When you call the `GetFirstPrice` function with a valid (existing) combination of Order ID and Product ID field values, the function returns the unit price from the found record. If the desired combination of field values can't be found in the table, the function returns **Null**.

If the current index is a multiple-field index, trailing key value can be omitted and are treated as **Null**. That is, you can leave off any number of key values from the end of a `Seek` method's *key* argument, but not from the beginning or middle.

Using Bookmarks Instead of Record Numbers

If you have used another database or programming environment, you may be accustomed to referring to record numbers. For example, you may have written code that opens a text file and thereafter refers to specific records by their relative position in the file. The first record in the file would be record 1, the second would be record 2, and so on.

In Microsoft Access, your view of records is usually a subset of the records in an entire table. Because the actual number of records can change at any time, especially in a multiuser environment, there's no absolute record number you can use to refer to a particular record. Instead, you can use bookmarks to identify and then return to a particular record.

A bookmark is a system-generated, randomly created string that uniquely identifies each record. To save a bookmark, you assign the value of the `Bookmark` property to a string variable. To return to the record, you set the `Bookmark` property to the value of the variable.

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The following example illustrates how you can use a bookmark to save the current record position and then quickly return to that record position if a find or seek operation fails. If the operation fails, the current record position is undefined.

```
Dim MyDb As Database, MyTable As Recordset, MyBookmark As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyTable = MyDb.OpenRecordset("Customers", DB_OPEN_TABLE)

MyTable.Index = "Company Name"
...
MyBookmark = MyTable.Bookmark
MyTable.Seek ">=", "Z"

If MyTable.NoMatch Then
    MsgBox "Can't find a company name starting with 'Z'."
    MyTable.Bookmark = MyBookmark
End If

MyTable.Close
MyDb.Close
```

In this example, saving the Bookmark property, and then resetting the Bookmark property to its previous value when the Seek method fails, causes the previously current record to be the current record again.

You can also use the Bookmark property for the dynaset used by a form. This lets your code mark which record is displayed on the form, and to change the record being displayed, by setting the form's Bookmark property. For example, you can add to a form containing employee information a button that a user can click to show the record for an employee's supervisor. The following example illustrates the function you can call from the button's Click event.

```
Function ShowSupervisor() As Integer
    Dim MyForm As Form, MySet As Recordset, MyBookmark As String

    Set MyForm = Screen.ActiveForm
    Set MySet = MyForm.Recordset.Clone
    MySet.Bookmark = MyForm.Bookmark

    MySet.FindFirst "[Employee ID] = " & MySet![Supervisor]
    If MySet.NoMatch Then
        MsgBox "Couldn't find employee's supervisor."
        MyForm.Bookmark = MyBookmark
    Else
        MyForm.Bookmark = MySet.Bookmark
    End If
End Function
```

```
MySet.Close
MyDb.Close
```

```
End Function
```

Some attached tables, for example, Paradox® tables that have no primary key, may not support bookmarks. Therefore, you can neither use bookmarks on such tables, nor use bookmarks on **Recordset** objects or queries based on those tables. However, all snapshots support bookmarks, regardless of their underlying tables. You can determine whether a table supports bookmarks by checking the value of the **Bookmarkable** property, as in the following example.

```
If MySet.Bookmarkable Then
    MsgBox "The underlying table supports bookmarks."
Else
    MsgBox "The underlying table doesn't support bookmarks."
End If
```

If you try to use bookmarks on a **Recordset** that doesn't support bookmarks, a run-time error occurs.

When you close a **Recordset**, any bookmarks you saved become invalid. You can't use a bookmark obtained from one **Recordset** in another **Recordset**, even if both **Recordset** objects are based on the same underlying table or query. A run-time error occurs if you try to share bookmarks between **Recordset** objects. However, you can use a bookmark on the duplicate (clone) of a **Recordset** as the following example shows.

```
Dim MyDb As Database
Dim OrigRecordset As Recordset, DupRecordset As Recordset
Dim Placeholder As String

Set MyDb = DBEngine.Workspaces(0).Databases(0)
' Create first Recordset.
Set OrigRecordset = MyDb.OpenRecordset("Orders",DB_OPEN_DYNASET)

Placeholder = OrigRecordset.Bookmark      ' Save current record position.
Set DupRecordset = OrigRecordset.Clone()  ' Create duplicate Recordset.
DupRecordset.Bookmark = Placeholder      ' Go to same record.
```

Changing Records

Once you've created a recordset variable of table or dynaset type, you can change, delete, or add new records. You can't change, delete, or add records to a snapshot-type **Recordset** object. This section presents the methods and procedures for changing data in table- and dynaset-type **Recordset** objects.

Modifying an Existing Record

Changing an existing record in a recordset variable is a four-step process:

- First, go to the desired record.
- Second, use the **Edit** method to prepare the current record for editing.
- Third, make the necessary changes to the record.
- Fourth, use the **Update** method to save the changes to the current record.

The following example illustrates how to change the job titles for all sales representatives in a table named Employees.

```
Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Employees")

MySet.MoveFirst
Do Until MySet.EOF
    If MySet![Title] = "Sales Representative" Then
        MySet.Edit
        MySet![Title] = "Account Executive"
        MySet.Update
    End If
    MySet.MoveNext
Loop

MySet.Close
MyDb.Close
```

If you don't use the **Edit** method before you try to change a value in the current record, a run-time error occurs.

Important If you change the current record, and then move to another record or close the **Recordset** without first using the **Update** method, your changes are lost without warning. For example, omitting the **Update** method from the preceding example results in no changes being made to the Employees table.

Dynaset-type **Recordset** objects can be based on a multiple-table query, with the query often implementing a one-to-many relationship. For example, suppose you want to create a multiple-table query that combines fields from the Orders and Order Details tables. Normally, you are prevented from changing values in the Orders table because it's on the "one" side of the relationship. Depending on your application, however, you may want to allow changes to the Orders table. To allow changes to the "one" side of a one-to-many relationship, you can use the **OpenRecordset** method's **DB_INCONSISTENT** constant to create an

inconsistent dynaset. This allows you to freely change values on the “one” side of the relationship.

```
Set MySet = MyDb.OpenRecordset("Sales Totals", DB_INCONSISTENT)
```

Important When you update an inconsistent dynaset, you can easily destroy the relational integrity of the data in the dynaset. You must take care to understand how the data is related across the one-to-many relationship and update the values on both sides in a way that preserves data integrity.

The `DB_INCONSISTENT` option is available only for dynaset-type **Recordset** objects. It's ignored for table- and snapshot-types.

Even with an inconsistent **Recordset**, some fields may not be updatable. For example, you can't change the value of a Counter field, and a **Recordset** based on certain attached tables may not be updatable.

Deleting an Existing Record

For more details on the **Delete** method, search Help for "Delete."

You can delete an existing record in a table- or dynaset-type **Recordset** using the **Delete** method. You can't delete records from a snapshot-type **Recordset**. The following example deletes all the records for trainees in a table named Employees.

```
Dim MyDb As Database, MySet As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Employees")

MySet.MoveFirst
Do Until MySet.EOF
    If MySet![Title] = "Trainee" Then
        MySet.Delete
    End If
    MySet.MoveNext
Loop

MySet.Close
MyDb.Close
```

When you use the **Delete** method, Microsoft Access immediately deletes the current record, without any warning or prompting. Deleting a record doesn't automatically cause the next record to become the current record; to move to the next record you must use the **MoveNext** method.

Adding a New Record

Adding a new record to a dynaset- or table-type **Recordset** is a three-step process:

- First, use the **AddNew** method to prepare a new record for editing.
- Second, assign values to each of the record's fields.
- Third, use the **Update** method to save the new record.

The following example adds a new record to a table named Shippers.

```
Dim MyDb As Database, MySet As Recordset

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MySet = MyDb.OpenRecordset("Shippers")

MySet.AddNew
MySet!["Company Name"] = "Global Parcel Service"
... ' Set remaining fields.
MySet.Update

MySet.Close
MyDb.Close
```

For additional information on the **AddNew** method, search Help for "AddNew."

When you use the **AddNew** method, Microsoft Access prepares a new, blank record and makes it the current record. When you use the **Update** method to save the new record, the record that was current before you used **AddNew** becomes the current record again. The new record's position in the **Recordset** depends on whether you added the record to a dynaset- or a table-type **Recordset**.

If you add a record to a dynaset-type **Recordset**, the new record appears at the end of the **Recordset**, no matter how the **Recordset** is sorted. To force the new record to appear in its properly sorted position, either recreate the **Recordset** or create a second **Recordset** based on the original.

If you add a record to a table-type **Recordset**, the record appears positioned according to the current index, or at the end of the table if there is no primary key and no current index.

Important If you use the **AddNew** method to add a new record, and then move to another record or close the **Recordset** without first using the **Update** method, your changes are lost without warning. For example, omitting the **Update** method from the preceding example results in no changes being made to the Shippers table.

Making Bulk Changes

Many of the changes you might otherwise perform in a loop can be done more efficiently with an update or delete query. For example, the task of changing all the job titles for sales representatives, illustrated in several previous examples by looping through each employee's record and inspecting the existing job title, can be quickly accomplished by an update query.

```
Dim MyDb As Database, MyQuery As QueryDef

Set MyDb = DBEngine.Workspaces(0).Databases(0)
Set MyQuery = MyDb.CreateQueryDef()
MyQuery.Name = "Change Job Titles"           ' Create query.
MyQuery.SQL = "UPDATE DISTINCTROW Employees ' Set SQL property.
↳ SET Employees!Title = 'Account Executive' WHERE Employees!Title =
↳ 'Sales Representative';"
MyDb.QueryDefs.Append MyQuery

MyQuery.Execute           ' Invoke query.
MyQuery.Close            ' Close query.
MyDb.QueryDefs.Delete MyQuery.Name ' Delete query.
```

Depending on your task, you may also find a delete query is more efficient than code that loops through records looking for records to delete.

Using Transactions to Control Changes

Often, to enforce data integrity, a set of operations must be considered as a single unit. For example, the transfer of funds from one bank account to another consists of two operations: entering a debit in one account and a matching credit in the other account. In practice, either both operations must succeed or neither operation should be processed.

In this situation, the debit and credit operations are a single unit, or a *transaction*. In Microsoft Access, a transaction is a set of operations that are *committed* (saved) if and only if all of the operations succeed. If any of the operations fails, all of the operations that succeeded are *rolled back* (canceled), and the data is returned to the state it was in before the transaction began.

Access Basic supports three transaction methods: one for starting, one for committing, and one for rolling back a transaction:

- The **BeginTrans** method begins a new transaction.
- The **CommitTrans** method commits all changes made to data since the most recent **BeginTrans** method.

- The **Rollback** method rolls back, or cancels, all changes made to data since the most recent **BeginTrans** method.

BeginTrans, **CommitTrans**, and **Rollback** are all methods of the **Workspace** object.

The following example changes the job title of all sales representatives in the **Employees** table of the current database. After the **BeginTrans** method starts a transaction that isolates all the changes made to the **Employees** table, the **CommitTrans** method saves the changes. Notice that you can use the **Rollback** method to undo changes that you saved with the **Update** method.

```
Function ChangeTitle
    Const MB_ICONQUESTION = 32
    Const YES = 6
    Const YES_NO = 4

    Dim CRLF As String, EmployeeName As String
    Dim Message As String, Prompt As String
    Dim MyDb As Database, MyWorkSpace as Workspace, MyTable As Recordset

    CRLF = Chr$(13) & Chr$(10)
    Prompt = "Change title to Account Executive?"

    ' Get current database.
    Set MyWorkspace = DBEngine.Workspaces(0)
    Set MyDb = MyWorkspace.Databases(0)
    ' Open table.
    Set MyTable = MyDb.OpenRecordset("Employees", DB_OPEN_TABLE)

    MyWorkspace.BeginTrans           ' Start of transaction scope.

    Do Until MyTable.EOF

        If MyTable![Title] = "Sales Representative" Then
            EmployeeName = MyTable![Last Name] & ", " & MyTable![First
                ↳ Name]
            Message = "Employee: " & EmployeeName & CRLF & CRLF
            If MsgBox(Message & Prompt, MB_ICONQUESTION & YES_NO,
                ↳ "Change Job Title") = YES Then

                MyTable.Edit           ' Enable
                                        ' editing.
                MyTable![Title] = "Account Executive" ' Change title.
                MyTable.Update         ' Save changes.
            End If
        End If
    End If
```

```

        MyTable.MoveNext           ' Move to next record.
    Loop

    If MsgBox("Save all changes?", MB_ICONQUESTION + YES_NO, "Save
    ↪ Changes") = YES Then
        MyWorkspace.CommitTrans   ' Commit changes.
    Else
        MyWorkspace.Rollback      ' Undo changes.
    End If

    MyTable.Close                 ' Close table.

End Function

```

Transactions and Performance

In addition to ensuring data integrity, transactions are important for improving database performance in many situations. Transactions provide speed increases because they tell the Jet database engine when it can flush memory caches to disk. Transactions bracket related operations so they can be cached in memory until either a **CommitTrans** or **Rollback** method occurs. Storing a set of operations in memory is much faster than writing every operation to disk, as occurs without transactions.

For more information on writing applications for use in a multiuser environment, see Chapter 12, "Developing Multiuser Applications."

Transactions are also a good way to increase *concurrency*—the number of users that can access a database simultaneously—in multiuser applications. For example, the following sequence allows more users to get at data simultaneously without locking conflicts:

```

MyWorkspace.BeginTrans
MyRs = MyDb.OpenRecordset("Long running query")
MyWorkspace.CommitTrans

Debug.Print MyRs!Field1
...

```

Computer Sample in CBC**CBC Symbols**

⠠⠠⠠	begin Computer Braille Code	⠠⠠	double quote
⠠⠠⠠	end Computer Braille Code	⠠⠠	single quote, apostrophe
⠠⠠	shift indicator	⠠⠠	equal sign
⠠⠠⠠	caps lock indicator	⠠⠠	greater than sign
⠠⠠⠠	caps release indicator	⠠⠠	less than sign
⠠⠠⠠	begin emphasis	⠠⠠	plus sign
⠠⠠⠠	end emphasis	⠠⠠	opening parenthesis
⠠⠠⠠	transcriber's option symbol: begin italic type within the Computer Braille Code	⠠⠠	closing parenthesis
⠠⠠⠠	termination indicator: end italic type within the Computer Braille Code	⠠⠠	opening bracket
⠠⠠⠠	continuation indicator	⠠⠠	closing bracket
⠠⠠	isolated lower-cell indicator	⠠⠠⠠	underscore
⠠⠠⠠	transcriber's note symbol	⠠⠠	ampersand
⠠⠠⠠	braille reference indicator; used to mark a marginal note	⠠⠠	exclamation point
⠠⠠	period	⠠⠠	semicolon
⠠⠠⠠⠠	ellipsis	⠠⠠	dollar sign
⠠⠠	comma	⠠⠠	question mark
		⠠⠠	colon

Computer Sample in UEBC

UEBC Symbols			
⠠⠠⠠	bold word	⠠⠠⠠	equal sign
⠠⠠⠠	begin bold passage	⠠⠠⠠	plus sign
⠠	(before a space)	⠠⠠⠠	greater than sign
⠠⠠⠠	end bold passage	⠠⠠⠠	less than sign
⠠⠠⠠	italic word	⠠⠠⠠	opening parenthesis
⠠⠠⠠	begin italic passage	⠠⠠⠠	closing parenthesis
⠠⠠⠠	end italic	⠠⠠⠠	opening bracket
⠠	(before a space)	⠠⠠⠠	closing bracket
⠠	(at end of line)	⠠⠠⠠	nondirectional double quote
⠠⠠⠠	continuation indicator	⠠	nondirectional single quote, apostrophe
⠠⠠⠠	(at end of line) continuation indicator with a space	⠠⠠⠠	underscore
⠠⠠⠠⠠	begin grade one passage	⠠⠠⠠	ampersand
⠠	(before a space)	⠠⠠⠠⠠	registered symbol (circled R)
⠠⠠⠠⠠	end grade one passage	⠠⠠⠠	bullet
⠠⠠⠠⠠	begin capitalized passage	⠠⠠⠠	dollar sign
⠠⠠⠠	end capitals within a sequence	⠠⠠⠠	dash
⠠	(before a space)		
⠠⠠⠠⠠	end capitalized passage		
⠠⠠⠠⠠	ellipsis		

Computer
The computer is a machine that can store and process information.
It can be used for many different purposes.
For example, it can be used to calculate numbers.
It can also be used to store data.
The computer is very useful.
It can help us to do our work more quickly.
It can also help us to learn more about the world.
The computer is a very important part of our lives.
It has changed the way we live and work.
Without the computer, many things would be impossible.
The computer is a wonderful invention.
It has made our lives so much easier.
We are very grateful for the computer.
It has truly revolutionized the world.
The computer is the future.
It will continue to make our lives better.
We look forward to the many things it will do for us.
The computer is a gift from the future.
We must take good care of it.
It is our responsibility to use it wisely.
The computer is a powerful tool.
It can do so much for us.
We must use it to its full potential.
The computer is a great asset.
It is something we should all be proud of.
The computer is the key to success.
It is the only way to stay ahead.
The computer is the heart of the modern world.
It is the engine of progress.
The computer is the future of humanity.
It is the light of the world.
The computer is the answer to all our problems.
It is the solution to all our needs.
The computer is the future of our children.
It is the future of our grandchildren.
The computer is the future of our world.
It is the future of our lives.
The computer is the future of our dreams.
It is the future of our hopes.
The computer is the future of our future.
It is the future of our future.
The computer is the future of our future.
It is the future of our future.

Computer
The computer is a machine that can do many things. It can store information, and it can do calculations very quickly. It can also help us to do things that we cannot do by ourselves. For example, it can help us to find out what the weather will be like tomorrow, or it can help us to find out what the best way to get to a certain place is. The computer is a very useful machine, and it is becoming more and more important in our lives.

There are many different kinds of computers. Some are very small, and some are very large. Some are used for business, and some are used for entertainment. Some are used for science, and some are used for education. The computer is a very versatile machine, and it can be used for many different purposes.

The computer is a very important part of our lives. It has changed the way we live, and it is continuing to change the way we live. It has made our lives easier, and it has made our lives more interesting. The computer is a very useful machine, and it is becoming more and more important in our lives.

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The computer is a machine that can do many things. It can store information, calculate numbers, and control other machines. It is used in many different ways, such as in schools, businesses, and homes. The computer is a very important part of our lives.

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Computer
The computer is a machine that can store information and perform operations on that information.
It can be used to control machinery, to calculate, and to communicate.
The computer is a very useful machine.
It is used in many different ways.
Some of the things that computers can do are:
- calculate numbers
- store information
- control machinery
- communicate
- draw pictures
- play games
- control traffic lights
- control the train system
- control the electricity supply
- control the telephone system
- control the internet
- control the weather forecast
- control the stock exchange
- control the bank
- control the government
- control the military
- control the police
- control the fire service
- control the ambulance service
- control the hospital
- control the school
- control the university
- control the factory
- control the office
- control the shop
- control the bank
- control the government
- control the military
- control the police
- control the fire service
- control the ambulance service
- control the hospital
- control the school
- control the university
- control the factory
- control the office
- control the shop

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It can help us to learn and to work. We can use a computer to find out about things we are interested in. We can use it to write letters and to make lists. We can use it to play games and to watch movies. The computer is a very useful tool.

There are many different kinds of computers. Some are very big and expensive, and some are very small and cheap. We can use a computer to do many different things. We can use it to learn and to work. We can use it to play games and to watch movies. The computer is a very useful tool.

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Braille text consisting of multiple lines of characters, including a minus sign and various Braille symbols.



Computer is a machine that can do many things. It can store information, calculate numbers, and control machines. It is used in many ways, from playing games to running businesses. Computers are becoming more and more important in our lives.

The computer is a very useful machine. It can do many things that people can't do by hand. It can store a lot of information, and it can find it very quickly. It can also do math very fast. Computers are used in many places, like schools, hospitals, and factories.

There are many different kinds of computers. Some are small and fit in your pocket, like a smartphone. Some are big and look like a desk, like a desktop computer. Some are even bigger, like the supercomputers that are used in science and research.

Computers are very important in our lives. They help us to work faster and better. They help us to learn new things and to have fun. Without computers, our lives would be very different.

Computers are used in many ways. They are used to play games, to watch movies, and to listen to music. They are also used to do school work, to run businesses, and to control machines. Computers are becoming more and more important in our lives.

1. The first part of the document discusses the importance of
 maintaining accurate records in a computer system. It
 emphasizes that data integrity is crucial for the
 reliability of the information processed. The document
 outlines various methods for ensuring data accuracy, such
 as regular backups and validation checks. It also
 discusses the role of user training in minimizing
 errors and the importance of clear documentation.

2. The second part of the document focuses on the
 security of computer systems. It highlights the risks
 of unauthorized access and data theft, and provides
 recommendations for implementing strong security
 protocols. This includes the use of firewalls, antivirus
 software, and secure communication channels. The
 document also addresses the need for regular security
 audits and updates to protect against emerging
 threats.

3. The third part of the document covers the topic of
 system performance and optimization. It discusses
 factors that can affect system speed and efficiency,
 such as hardware configuration and software design.
 The document provides strategies for identifying
 performance bottlenecks and offers solutions for
 optimizing system resources. It also mentions the
 importance of monitoring system health and
 performance metrics.

4. The final part of the document discusses the
 future of computer technology and its impact on
 society. It explores emerging trends such as
 artificial intelligence, cloud computing, and the
 Internet of Things. The document also addresses
 ethical considerations and the need for responsible
 technology use. It concludes by emphasizing the
 ongoing nature of computer science and the
 importance of staying current in this rapidly
 evolving field.

Computer is a device that can store and process information.
It can perform a large number of calculations very quickly.
It can also store information for a long time.
The computer is used in many different ways.
For example, it can be used to control a machine.
It can also be used to communicate with other computers.
The computer is a very important part of our lives.
It has changed the way we live and work.
Without computers, many of the things we do today
would be impossible. Computers are used in schools,
business, and government. They are also used in
medicine and science. The computer is a very
powerful tool that has many uses. It is one of
the most important inventions of the 20th century.
The computer has made it possible for us to do
things that we could not do before. It has
made our lives easier and more convenient.
The computer is a very useful tool that we
cannot live without. It is a part of our
everyday lives. The computer is a very
important part of our lives. It has
changed the way we live and work.

Computer is a machine that can store and process information. It can be used for many different things, like keeping records, doing calculations, and controlling machines.

There are many different kinds of computers. Some are very big and expensive, and some are very small and cheap.

Some computers are used for business, and some are used for education. Some are used for entertainment, and some are used for science.

Computers are very useful. They can help us do things that we could not do without them.

Computers are used in many different ways. They can be used to control machines, to store information, and to do calculations.

Computers are used in many different places. They are used in schools, in businesses, in homes, and in factories.

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Computers

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本专业学生主要学习计算机组成原理、操作系统、数据库系统、网络工程、人工智能等课程。通过本课程的学习，使学生具有扎实的计算机科学与技术理论基础，具有良好的工程实践能力，能够在企事业单位从事计算机科学与技术工作的复合型、应用型高级专门人才。

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1. The computer is a very important part of our life. It can help us do many things. For example, we can use it to write letters, to draw pictures, and to play games. It is very useful.

2. In the past, computers were very big and expensive. They were only used in big offices and schools. But now, computers are getting smaller and cheaper. We can find them everywhere.

3. There are many kinds of computers. Some are very fast and can do a lot of work. Some are very slow and can only do simple things. We should choose the right one for our needs.

4. The computer is changing our life. We can use it to learn, to work, and to have fun. It is a great invention. We should use it wisely.

5. In the future, computers will be even more important. They will help us solve many problems. We should continue to study and improve them.

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Computer
is a device that can be programmed
to carry out sequences of
arithmetic or logical operations
automatically and rapidly, without
the need for human intervention
for each operation. The operations
are carried out by a central
processing unit (CPU) which
is connected to various input
and output devices. The CPU
is the brain of the computer
and is responsible for
controlling the flow of data
and instructions. It consists
of the central processing unit
and the memory system. The
memory system is used to store
data and instructions. The
input and output devices are
used to communicate with the
user. The computer is a
very powerful tool and is
used in many different
applications. It can be used
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Computer
The computer is a machine that can do many things. It can store information, calculate numbers, and control other machines. It is used in many different ways, such as in schools, businesses, and homes. The computer is a very important part of our lives.

Computer
The computer is a machine that can do many things. It can store information, and it can do calculations. It can also control other machines. For example, it can control a factory or a train. The computer is very useful, and it is becoming more and more important in our lives.

There are many different kinds of computers. Some are very small, and some are very large. Some are used for business, and some are used for entertainment. The computer is a very versatile machine, and it can be used for many different purposes.

The computer is a very important part of our lives. It has changed the way we live, and it will continue to change the way we live in the future. We should be proud of the computer, and we should use it wisely.

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Computer programming is the process of writing a set of instructions that can be followed by a computer to perform a task.

The instructions are written in a language that the computer can understand, such as C++ or Java.

Computer programming is a creative process that involves designing algorithms and writing code that can be executed by a computer.

Computer programming is a profession that requires a deep understanding of computer science and programming languages.

Computer programming is a challenging and rewarding career choice.

Computer programming is a profession that offers a variety of opportunities for growth and advancement.

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1. The computer is a very useful tool. It can help us in many ways. For example, it can help us to do our homework. We can use it to look up information on the Internet. We can also use it to play games and watch movies. The computer is a very important part of our lives. We should use it wisely and not spend too much time on it.

2. The computer is a very useful tool. It can help us in many ways. For example, it can help us to do our homework. We can use it to look up information on the Internet. We can also use it to play games and watch movies. The computer is a very important part of our lives. We should use it wisely and not spend too much time on it.

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1. The computer is a very important part of our lives. It helps us in many ways.

2. We use computers for work, school, and entertainment.

3. Computers are used in many different fields.

4. They are used in science, medicine, and business.

5. Computers help us to communicate with each other.

6. They make our lives easier and more convenient.

7. Without computers, our world would be very different.

8. Computers are everywhere.

9. They are used in almost every aspect of our lives.

10. Computers have changed the way we live and work.

11. They have made our lives more productive and efficient.

12. Computers are used in many different ways.

13. They are used in education, science, and industry.

14. Computers help us to learn and grow.

15. They are used in many different ways.

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Sample 6. Chemistry

This sample is transcribed using *Braille Code for Chemical Notation 1997* (on left hand pages) and the Unified English Braille Code as of June 2001 (on right hand pages).

In the UEBC sample, chain and ring diagrams are transcribed using grade one mode. Runover locators in these diagrams are preceded by an **r**.



Sugars, starch, and cellulose are all carbohydrates.

Every moment of your life, carbohydrates are oxidized in your body to produce energy, CO_2 , and H_2O .

B.3 CARBOHYDRATES: THE ENERGIZERS

Carbohydrates are compounds composed of only three elements—carbon, hydrogen, and oxygen. For example, glucose, the key energy-releasing carbohydrate in biological systems, has the formula $C_6H_{12}O_6$. When such formulas were first discovered, chemists were tempted to write the glucose formula as $C_6(H_2O)_6$ —implying a chemical combination of carbon and water. So, they invented the term “carbo-hydrate,” or water-containing carbon compound. We now know that water molecules are not actually present in carbohydrates, but the name has persisted.

Carbohydrates may be simple sugars such as glucose or composed of two or more simple sugar molecules combined in various ways (Table 3). Simple sugars are called **monosaccharides**. The most common monosaccharide molecules contain five or six carbon atoms bonded together. As shown in Figure 2, glucose (and most other monosaccharides) can exist either in a chain or a ring form. Do both forms have the same molecular formula? (Check by counting the atoms.)

Figure 2 Structural formulas for glucose. The chain and ring forms are interconvertible; the ring form predominates in the body.

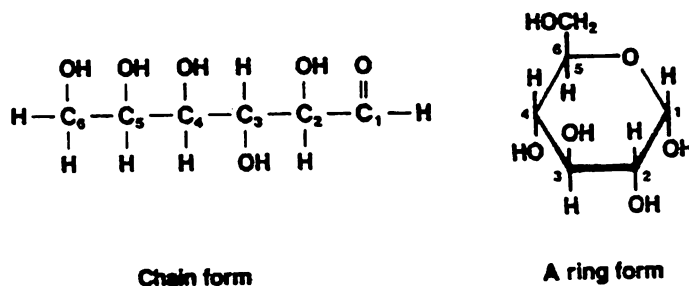
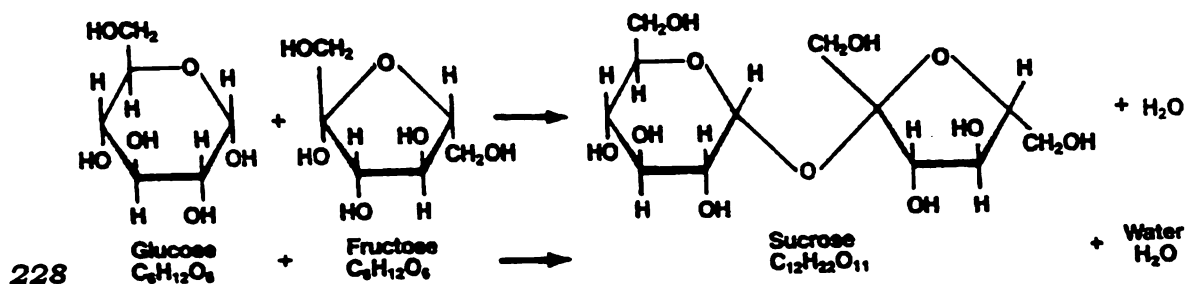
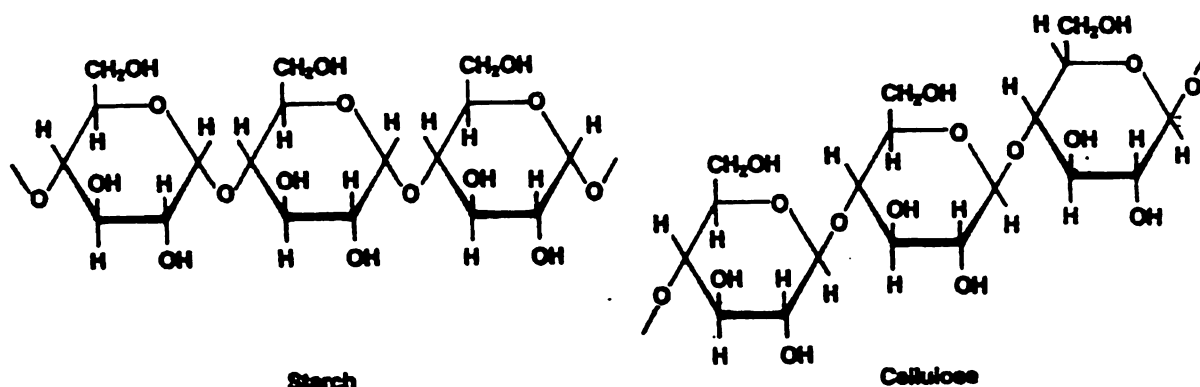


Figure 3 Formation of sucrose. The two shaded $-OH$ groups react with the elimination of one H_2O molecule.





Sugar molecules composed of two simple sugar units are called **disaccharides**. Sucrose ($C_{12}H_{22}O_{11}$ —ordinary table sugar) is a disaccharide in which the ring forms of glucose and fructose are joined (see Figure 3). As the molecular structures suggest, monosaccharides and disaccharides are composed of polar molecules. Thus they tend to be highly soluble in water, a polar solvent.

Polymers composed of units of simple sugar molecules are called **polysaccharides** (Figure 4). Starch, a major component of grains and many vegetables, is a polysaccharide composed of glucose units. Cellulose, the fibrous or woody material of plants and trees, is another polysaccharide formed from glucose. The types of carbohydrates are summarized in Table 3.

Figure 4 Polysaccharides. Starch and cellulose are both polymers of glucose. They differ in the arrangements of the bonds that join the glucose monomers.

Recall the "like dissolves like" rule.

Remember the discussion of polyethylene on page 201. A polymer is a large molecule composed of many smaller molecular units chemically bonded together.

Table 3 CARBOHYDRATES

Classification and Examples	Composition	Formula	Common Name or Source
Monosaccharides		$C_6H_{12}O_6$	
Glucose	—		Blood sugar
Fructose	—		Fruit sugar
Galactose	—		—
Disaccharides		$C_{12}H_{22}O_{11}$	
Sucrose	Fructose + glucose		Cane sugar
Lactose	Galactose + glucose		Milk sugar
Maltose	Glucose + glucose		Germinating seeds
Polysaccharides	Glucose polymers	—	
Starch			Plants
Glycogen			Animals
Cellulose			Fiber

Carbohydrates are all sugars or polymers of sugars.

During photosynthesis, green plants produce glucose. The overall reaction is as follows:



Plants build these glucose molecules either into starch for energy storage, or into cellulose, becoming part of the plant's structure.

There are exceptions—cows and termites, for example, can digest cellulose.

1 g carbohydrate = 4 Cal energy

Sugars and starch are rapidly digested in your body, making them convenient sources of energy. Cellulose is not digested. The glucose units in cellulose are bonded together differently than they are in starch (see Figure 4). The slight difference in bonding makes cellulose indigestible by most animals. However, cellulose, sometimes called fiber, is needed in the diet to keep the digestive system functioning properly.

Sugars and starch are the major energy-delivering substances in our diets. Even the smallest muscle twitch or thought requires energy. The body obtains most of this energy by burning the glucose molecules in sugars and starch. Each gram of carbohydrate delivers about 4 Calories of energy.

Nutritionists recommend that about 60% of food energy come from carbohydrates. Most of the world's population obtains carbohydrates by eating grains. These grains are often consumed as rice, corn meal, wheat tortillas, bread, and pasta. In the United States we tend to eat more wheat breads and potatoes for carbohydrates than people do elsewhere. In all countries, fruits and vegetables also provide carbohydrates. Meats provide a small amount of carbohydrate in the form of glycogen, which is how animals store glucose. On average, each U.S. citizen consumes more than 90 lb (40 kg) of table sugar each year in beverages, breads, and cakes and as a sweetener. A 12-ounce non-diet cola drink contains nine teaspoons of sugar.

B.4 FATS: STORED ENERGY WITH A BAD NAME

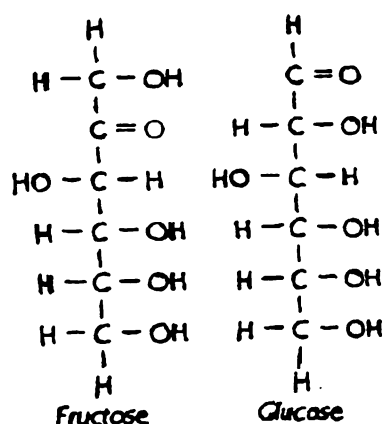
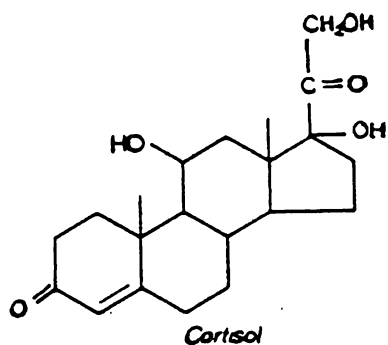
Unlike carbohydrate and protein, the word "fat" has acquired its own general (and somewhat negative) meaning. To most people, "You're too fat" means that the person looks overweight. However, from a chemist's point of view, **fats** are a major category of biomolecules which have their own special characteristics and functions, just as carbohydrates and proteins do.

Fats are a significant part of our diet. They're present in meat, fish, and poultry; salad dressings and oils; dairy products; and grains. When our bodies take in more food than is needed for energy, much of the excess is converted to fat molecules and stored in the body. If food intake is not large enough to supply the body's energy needs, the body begins to burn stored fat.

Like carbohydrates, fats are composed of carbon, hydrogen, and oxygen. However, fats contain less oxygen than carbohydrates and contain more stored energy. Gram-for-gram, fat contains over twice the energy found in carbohydrates. Fats are generally nonpolar in nature and are only sparingly soluble in water. Because of their solubility and energy-storing properties, fat molecules are more like hydrocarbons than carbohydrates.

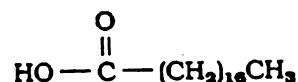
Fats are members of the class of biomolecules called **lipids**. Some lipids are builder molecules that form cell membranes. Others become hormones—chemical messengers that regulate processes in the body.

A typical fat molecule is a combination of a simple three-carbon alcohol called **glycerol** with three fatty acid molecules. (The formation of a typical fat is shown in Figure 5.) **Fatty acids** are a class of compounds made up of a long hydrocarbon chain with a carboxyl group (—COOH) at one end. The reaction producing a fat molecule is similar to one you already completed in the laboratory (page 204)—the production of an ester, methyl

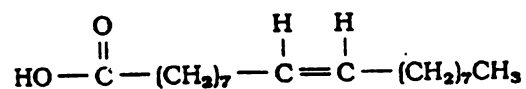


A molecule may contain more than one functional group. For example, look at the structure of cortisol (a lipid) in the margin. Cortisol is a hormone which, when released during starvation, makes it possible to use energy from protein. Note that cortisol contains several functional groups: three —OH groups, two C=O groups, and one C=C double bond.

1. Refer to the two forms of glucose shown in Figure 2.
 - a. Draw both chain and ring structures for glucose.
 - b. Circle and identify the functional group(s) found in the chain structure.
 - c. Examine the numbering of the carbon atoms and the functional groups attached to each carbon atom.
 - (1) Which functional groups apparently react to form the ring structure?
 - (2) On which carbon atoms do these functional groups appear?
2. Compare the straight-chain structures of fructose and glucose (See margin.) Describe the differences in the structures of these two monosaccharides.
3. In general, alcohols react with organic (carboxylic) acids to form esters. Using the equation shown in Figure 5 as a guide, write an equation (including structures) for the reaction of stearic acid (a fatty acid) with glycerol to form glyceryl tristearate (a fat). Stearic acid has this structural formula:



4. Copy the following molecular structure on your own paper:



- a. Circle and identify the functional group(s).
- b. Is this molecule a carbohydrate or a fatty acid? Why?
- c. Is it saturated or unsaturated? Why?
- d. Rewrite the molecular structure to show the carbon atoms in a continuous chain.

Saturated fats appear to contribute to coronary heart disease.

The term **polyunsaturated**, often used in food advertising, means that the food contains fats with two or more carbon-carbon double bonds in each fatty acid molecule. The term has become familiar because increasing evidence suggests that saturated fats may contribute to health problems, while some unsaturated fats may not. Saturated fats are associated with formation of plaque (fat-like or fibrous matter), which can block arteries. The result is a condition known as "hardening of the arteries," or atherosclerosis, a particular threat to coronary (heart) arteries and arteries leading to the brain. If coronary arteries are blocked, a heart attack can result, damaging the heart muscle.

Chemistry Sample in Chemistry Code

Nemeth Symbols

⠆	baseline indicator	⠆	return to baseline after subscript
⠆	decimal point	⠆	mathematical comma
⠆⠆	general reference indicator denoting marginal material	⠆	plus sign
⠆	punctuation indicator	⠆⠆	right arrow
⠆	opening parenthesis	⠆⠆	transcriber's note symbol
⠆	closing parenthesis	⠆⠆	percent sign
⠆	begin subscript		

Chemistry Code Symbols

⠆⠆⠆	single horizontal bond	⠆⠆	lower left to upper right oblique bond
⠆⠆⠆	bold horizontal bond	⠆⠆	upper left to lower right oblique bond
⠆⠆⠆	double horizontal bond	⠆⠆⠆	lower left to upper right, double, oblique bond
⠆	single vertical bond	⠆⠆	up-pointing, filled in, oblique wedge
⠆⠆	double vertical bond		
⠆⠆	unlabeled vertex		

Chemistry Sample in UEBC

UEBC Symbols

⠠⠠⠠⠠	begin capitalized passage	⠠⠠	dash
⠠	(before a space)	⠠⠠	subscript next item
⠠⠠⠠⠠	end capitalized passage	⠠⠠⠠	opening parenthesis
⠠⠠⠠	italic word	⠠⠠⠠	closing parenthesis
⠠⠠⠠	begin italic passage	⠠⠠⠠	single horizontal bond
⠠	(before a space)	⠠⠠⠠⠠	double horizontal bond
⠠⠠⠠	end italic passage	⠠⠠⠠	plus sign
⠠	grade one symbol	⠠⠠⠠	equal sign
⠠⠠⠠	grade one word	⠠⠠⠠	right arrow
⠠⠠⠠⠠	begin grade one passage	⠠⠠⠠	transcriber's note symbol
⠠⠠⠠	end grade one within a sequence	⠠⠠⠠	percent sign
⠠	(before a space)		
⠠⠠⠠	end grade one passage		

Line Symbols in UEBC

⠠	single vertical line	⠠	single left-leaning diagonal line
⠠⠠	double vertical line	⠠⠠	double right-leaning diagonal line
⠠⠠⠠	single horizontal line	⠠	crossing with vertical line
⠠	single right-leaning diagonal line	⠠	right corner with upward vertical

Note: UEBC does not yet provide for the bold line or for the wedge.

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1. The reaction of sodium metal with water is highly exothermic and produces hydrogen gas. The reaction is represented by the following balanced chemical equation:

$$2\text{Na} + 2\text{H}_2\text{O} \rightarrow 2\text{NaOH} + \text{H}_2$$

2. In this reaction, sodium metal is oxidized to sodium hydroxide, and water is reduced to hydrogen gas. The half-reactions are:

$$\text{Na} \rightarrow \text{Na}^+ + \text{e}^-$$
$$2\text{H}_2\text{O} + 2\text{e}^- \rightarrow \text{H}_2 + 2\text{OH}^-$$

3. The standard enthalpy of formation for sodium hydroxide is -425.8 kJ/mol . The standard enthalpy of formation for water is -285.8 kJ/mol . The standard enthalpy of formation for sodium metal is 0 kJ/mol , and for hydrogen gas is 0 kJ/mol .

4. The reaction is exothermic, as indicated by the negative value of the standard enthalpy of formation for sodium hydroxide. The reaction releases heat to the surroundings.

5. The reaction is also exothermic, as indicated by the negative value of the standard enthalpy of formation for water. The reaction releases heat to the surroundings.

6. The reaction is exothermic, as indicated by the negative value of the standard enthalpy of formation for sodium hydroxide. The reaction releases heat to the surroundings.

7. The reaction is exothermic, as indicated by the negative value of the standard enthalpy of formation for sodium hydroxide. The reaction releases heat to the surroundings.

8. The reaction is exothermic, as indicated by the negative value of the standard enthalpy of formation for sodium hydroxide. The reaction releases heat to the surroundings.

9. The reaction is exothermic, as indicated by the negative value of the standard enthalpy of formation for sodium hydroxide. The reaction releases heat to the surroundings.

1. The reaction between sodium hydroxide and hydrochloric acid is an example of a neutralization reaction. The products are sodium chloride and water. The balanced chemical equation for this reaction is:

$$\text{NaOH} + \text{HCl} \rightarrow \text{NaCl} + \text{H}_2\text{O}$$

2. The reaction between sodium hydroxide and sulfuric acid is an example of a neutralization reaction. The products are sodium sulfate and water. The balanced chemical equation for this reaction is:

$$2\text{NaOH} + \text{H}_2\text{SO}_4 \rightarrow \text{Na}_2\text{SO}_4 + 2\text{H}_2\text{O}$$

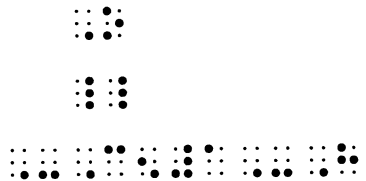
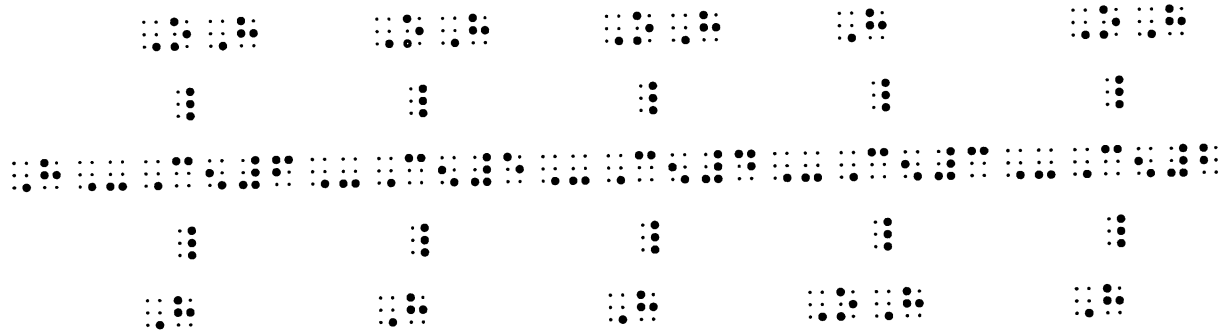
3. The reaction between sodium hydroxide and phosphoric acid is an example of a neutralization reaction. The products are sodium phosphate and water. The balanced chemical equation for this reaction is:

$$3\text{NaOH} + \text{H}_3\text{PO}_4 \rightarrow \text{Na}_3\text{PO}_4 + 3\text{H}_2\text{O}$$

4. The reaction between sodium hydroxide and acetic acid is an example of a neutralization reaction. The products are sodium acetate and water. The balanced chemical equation for this reaction is:

$$\text{NaOH} + \text{CH}_3\text{COOH} \rightarrow \text{CH}_3\text{COONa} + \text{H}_2\text{O}$$

5. The reaction between sodium hydroxide and carbonic acid is an example of a neutralization reaction. The products are sodium carbonate and water. The balanced chemical equation for this reaction is:



6. The reaction between sodium hydroxide and carbonic acid is an example of a neutralization reaction. The products are sodium bicarbonate and water. The balanced chemical equation for this reaction is:

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2. The following reaction is a redox reaction. Write the balanced half-reactions for the reaction.

$$\text{Fe} + \text{Cu}^{2+} \rightarrow \text{Fe}^{2+} + \text{Cu}$$

3. The following reaction is a redox reaction. Write the balanced half-reactions for the reaction.

$$\text{Zn} + \text{Pb}^{2+} \rightarrow \text{Zn}^{2+} + \text{Pb}$$

4. The following reaction is a redox reaction. Write the balanced half-reactions for the reaction.

$$\text{Al} + \text{Cu}^{2+} \rightarrow \text{Al}^{3+} + \text{Cu}$$

5. The following reaction is a redox reaction. Write the balanced half-reactions for the reaction.

$$\text{Mg} + \text{Ni}^{2+} \rightarrow \text{Mg}^{2+} + \text{Ni}$$

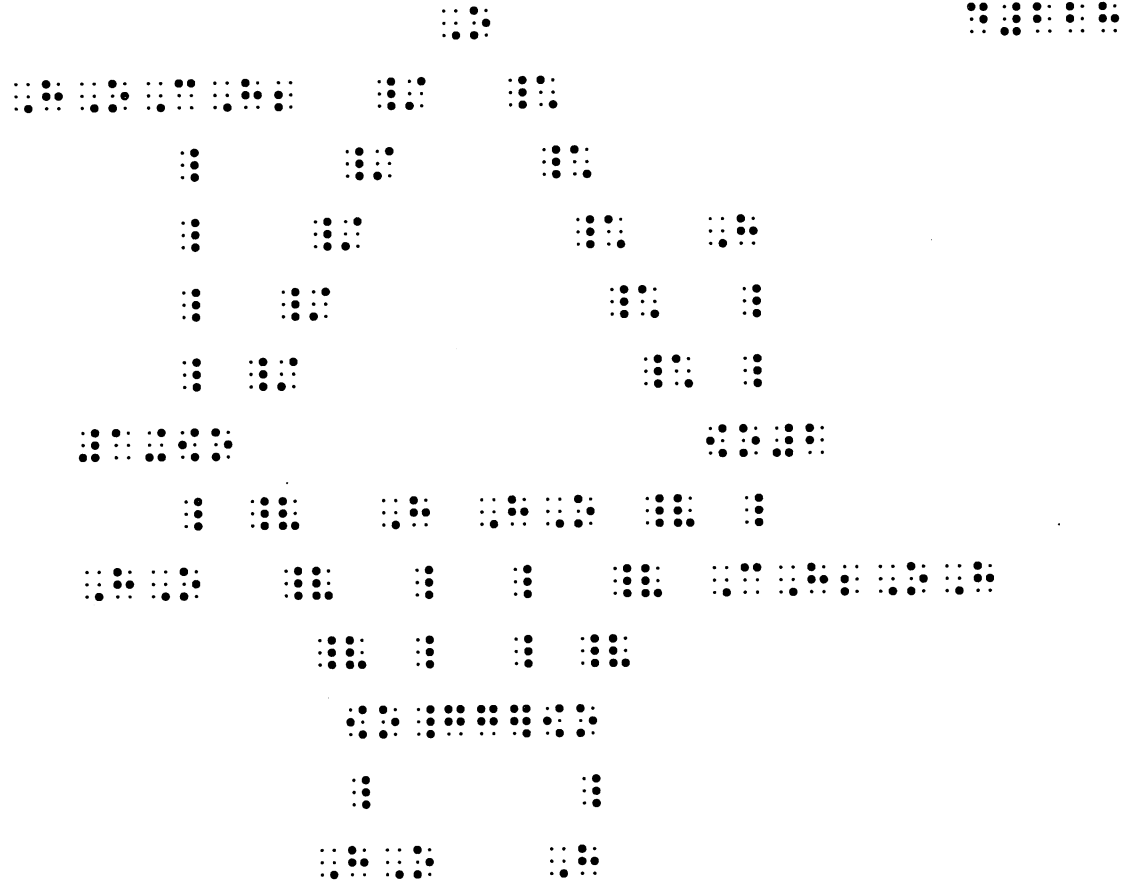
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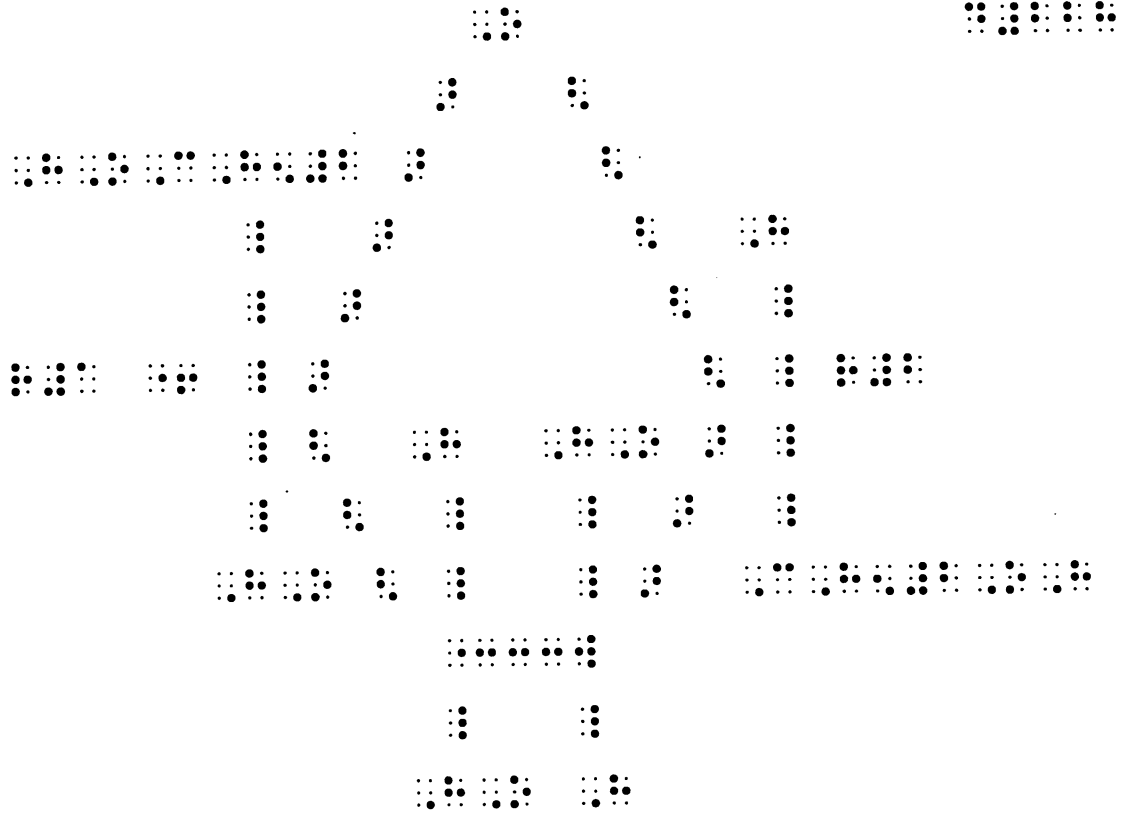
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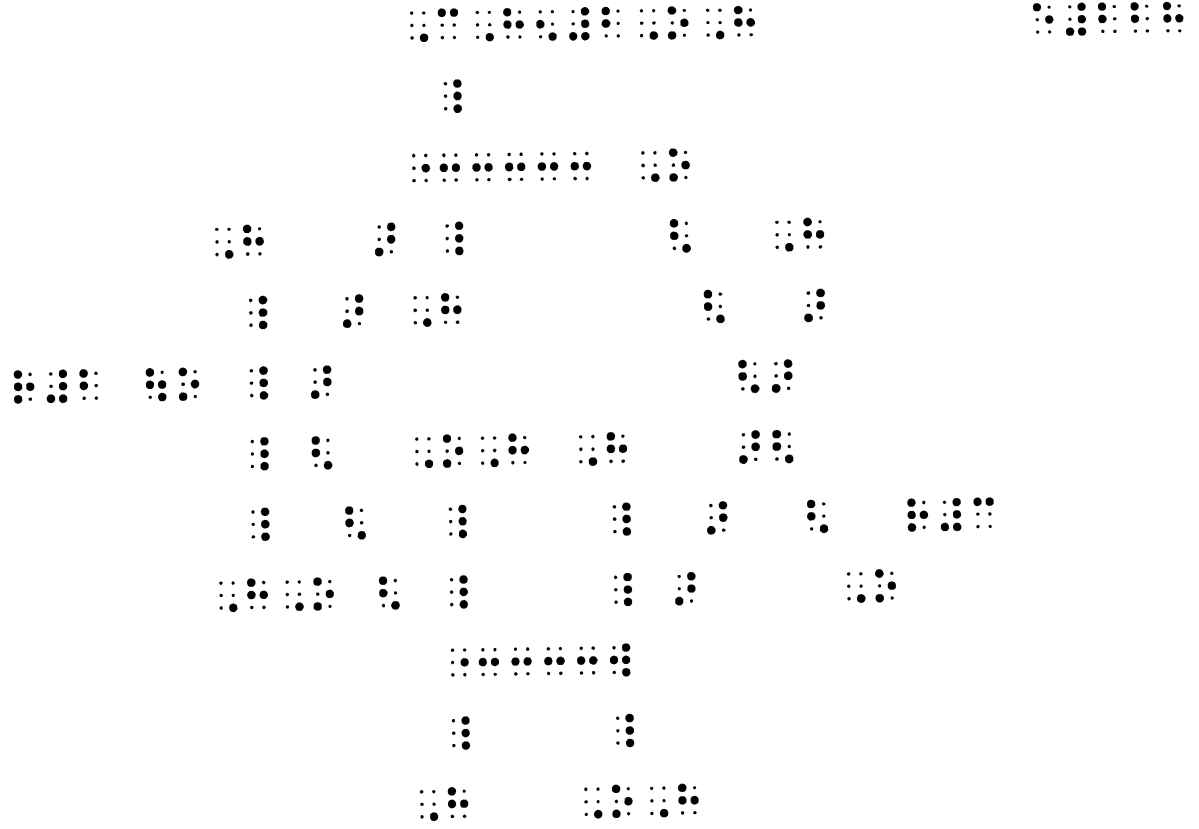
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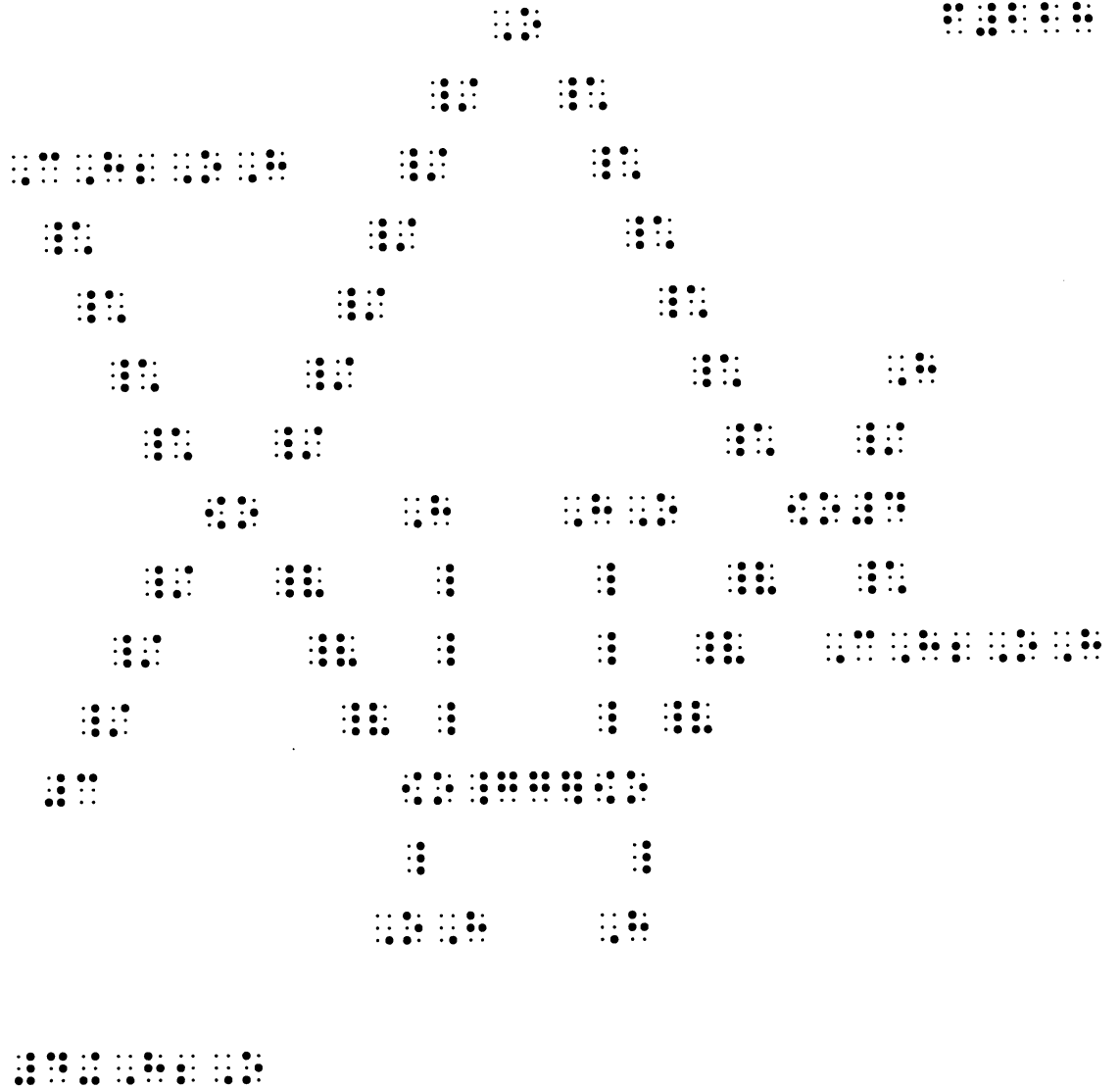
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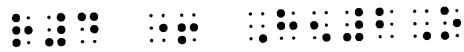
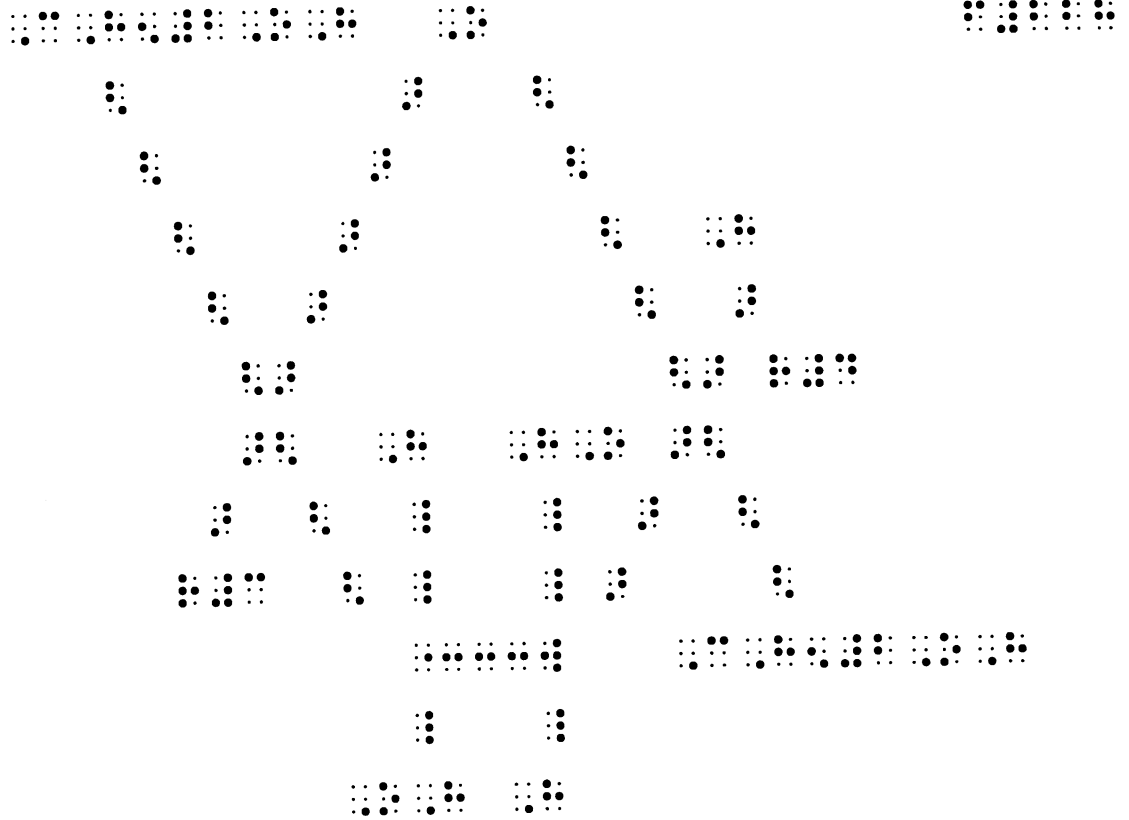
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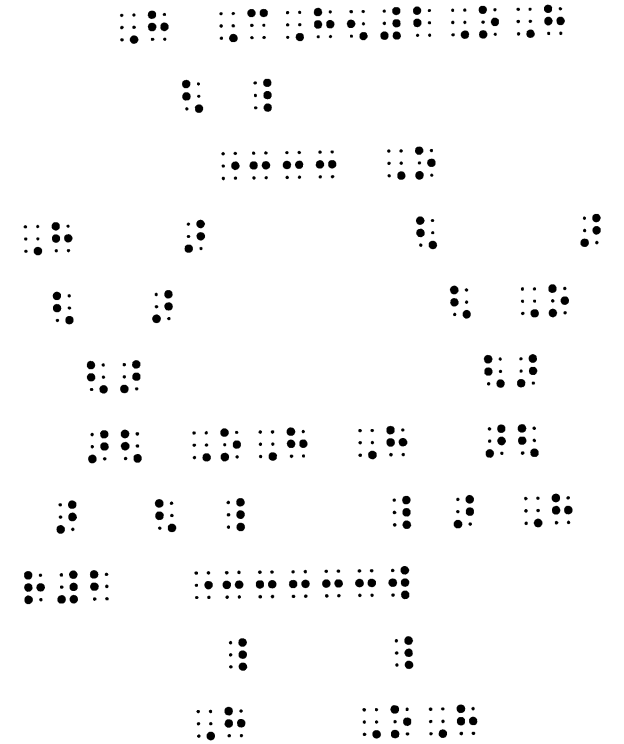
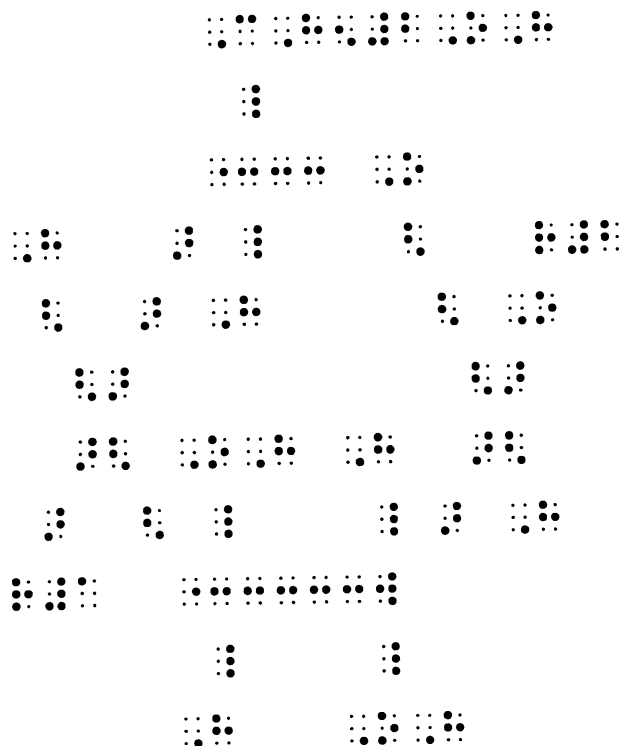
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2.000 mol of H_2 and 1.000 mol of O_2 are placed in a closed container at 25°C and 1.00 atm. The mixture is ignited and the reaction goes to completion. The final temperature is 25°C and the final pressure is 1.00 atm. Calculate the final volume of the gas mixture.

$\text{H}_2(\text{g}) + \frac{1}{2}\text{O}_2(\text{g}) \rightarrow \text{H}_2\text{O}(\text{l})$

Initial moles: $\text{H}_2 = 2.000$, $\text{O}_2 = 1.000$, $\text{H}_2\text{O} = 0$
Change in moles: $\text{H}_2 = -2.000$, $\text{O}_2 = -1.000$, $\text{H}_2\text{O} = +2.000$
Final moles: $\text{H}_2 = 0$, $\text{O}_2 = 0$, $\text{H}_2\text{O} = 2.000$

Initial volume: $V_i = n_i \frac{RT}{P} = (2.000 + 1.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 40.0 \text{ L}$

Final volume: $V_f = n_f \frac{RT}{P} = (2.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 20.0 \text{ L}$

Final volume of the gas mixture is 20.0 L.

2.000 mol of H_2 and 1.000 mol of O_2 are placed in a closed container at 25°C and 1.00 atm. The mixture is ignited and the reaction goes to completion. The final temperature is 25°C and the final pressure is 1.00 atm. Calculate the final volume of the gas mixture.

Initial moles: $\text{H}_2 = 2.000$, $\text{O}_2 = 1.000$, $\text{H}_2\text{O} = 0$
Change in moles: $\text{H}_2 = -2.000$, $\text{O}_2 = -1.000$, $\text{H}_2\text{O} = +2.000$
Final moles: $\text{H}_2 = 0$, $\text{O}_2 = 0$, $\text{H}_2\text{O} = 2.000$

Initial volume: $V_i = n_i \frac{RT}{P} = (2.000 + 1.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 40.0 \text{ L}$

Final volume: $V_f = n_f \frac{RT}{P} = (2.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 20.0 \text{ L}$

Final volume of the gas mixture is 20.0 L.

Initial moles: $\text{H}_2 = 2.000$, $\text{O}_2 = 1.000$, $\text{H}_2\text{O} = 0$
Change in moles: $\text{H}_2 = -2.000$, $\text{O}_2 = -1.000$, $\text{H}_2\text{O} = +2.000$
Final moles: $\text{H}_2 = 0$, $\text{O}_2 = 0$, $\text{H}_2\text{O} = 2.000$

Initial volume: $V_i = n_i \frac{RT}{P} = (2.000 + 1.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 40.0 \text{ L}$

Final volume: $V_f = n_f \frac{RT}{P} = (2.000) \frac{(0.08206 \text{ L}\cdot\text{atm}) (298.15 \text{ K})}{1.00 \text{ atm}} = 20.0 \text{ L}$

Final volume of the gas mixture is 20.0 L.

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The following table shows the results of the experiment. The first column shows the concentration of the reactants, the second column shows the initial rate of reaction, and the third column shows the order of reaction with respect to each reactant. The overall order of reaction is given in the fourth column.

Concentration of A (mol dm ⁻³)	Concentration of B (mol dm ⁻³)	Initial rate of reaction (mol dm ⁻³ s ⁻¹)	Order of reaction with respect to A	Order of reaction with respect to B	Overall order of reaction
0.10	0.10	0.010	1	1	2
0.20	0.10	0.040	2	0	2
0.10	0.20	0.020	0	1	1
0.20	0.20	0.040	1	1	2

From the table, it can be seen that the reaction is second order with respect to A and first order with respect to B. The overall order of reaction is therefore second order.

The rate equation for the reaction is:

$$\text{rate} = k[A]^2[B]$$

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Chemistry is the study of matter and the changes it undergoes. Matter is anything that has mass and takes up space. It can be a solid, liquid, or gas. Matter is made of atoms and molecules. Atoms are the smallest particles of matter that cannot be created or destroyed. Molecules are made of two or more atoms joined together. Matter can change from one state to another. For example, ice can melt to become water, and water can evaporate to become steam. These changes are called physical changes. In a physical change, the substance remains the same, but its form or state changes. In a chemical change, the substance changes into a new substance with different properties. For example, when iron reacts with oxygen, it forms iron oxide. In a chemical change, the atoms of the reactants are rearranged to form the products. Matter is conserved in a chemical change. The total mass of the reactants is equal to the total mass of the products. This is known as the law of conservation of mass. Matter is made of particles that are constantly moving. The particles in a solid are packed closely together and vibrate. The particles in a liquid are more loosely packed and can move past each other. The particles in a gas are far apart and move rapidly in all directions. The temperature of a substance affects the movement of its particles. As the temperature increases, the particles move faster and the substance expands. As the temperature decreases, the particles move slower and the substance contracts. Matter is made of atoms and molecules. Atoms are the smallest particles of matter that cannot be created or destroyed. Molecules are made of two or more atoms joined together. Matter can change from one state to another. For example, ice can melt to become water, and water can evaporate to become steam. These changes are called physical changes. In a physical change, the substance remains the same, but its form or state changes. In a chemical change, the substance changes into a new substance with different properties. For example, when iron reacts with oxygen, it forms iron oxide. In a chemical change, the atoms of the reactants are rearranged to form the products. Matter is conserved in a chemical change. The total mass of the reactants is equal to the total mass of the products. This is known as the law of conservation of mass. Matter is made of particles that are constantly moving. The particles in a solid are packed closely together and vibrate. The particles in a liquid are more loosely packed and can move past each other. The particles in a gas are far apart and move rapidly in all directions. The temperature of a substance affects the movement of its particles. As the temperature increases, the particles move faster and the substance expands. As the temperature decreases, the particles move slower and the substance contracts.

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2.000 mol of CaCl_2 is added to 1.000 mol of Na_2CO_3 .
The reaction is:

$\text{CaCl}_2 + \text{Na}_2\text{CO}_3 \rightarrow \text{CaCO}_3 + 2\text{NaCl}$

1.000 mol of CaCl_2 reacts with 1.000 mol of Na_2CO_3 to produce 1.000 mol of CaCO_3 and 2.000 mol of NaCl .

1.000 mol of CaCl_2 reacts with 1.000 mol of Na_2CO_3 to produce 1.000 mol of CaCO_3 and 2.000 mol of NaCl .

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1.000 mol of CaCl_2 reacts with 1.000 mol of Na_2CO_3 to produce 1.000 mol of CaCO_3 and 2.000 mol of NaCl .

2.00 mol of CaCl_2 is dissolved in 1.00 L of water. The resulting solution is diluted to a total volume of 2.00 L. What is the molarity of the resulting solution?

1.00 M
0.500 M
0.250 M
0.125 M

0.0625 M
0.0312 M
0.0156 M
0.00781 M

0.0312 M
0.0156 M
0.00781 M
0.00391 M

0.0156 M
0.00781 M
0.00391 M
0.00195 M

0.00976 M
0.00488 M
0.00244 M
0.00122 M

0.00616 M
0.00308 M
0.00154 M
0.00077 M

0.00308 M
0.00154 M
0.00077 M
0.000385 M

0.00154 M
0.00077 M
0.000385 M
0.000192 M

0.00077 M
0.000385 M
0.000192 M
0.000096 M

0.00488 M
0.00244 M
0.00122 M
0.00061 M

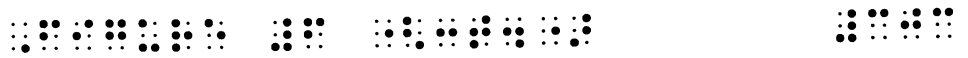
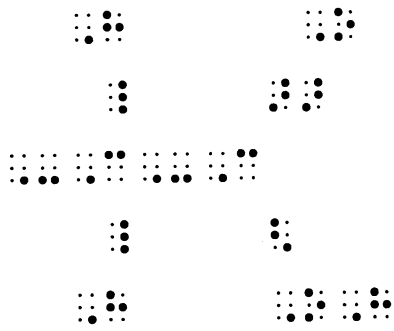
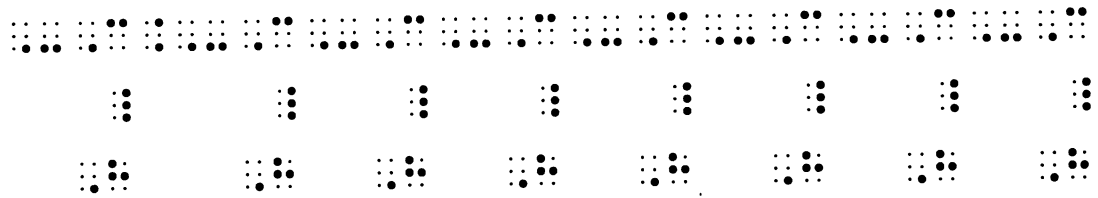
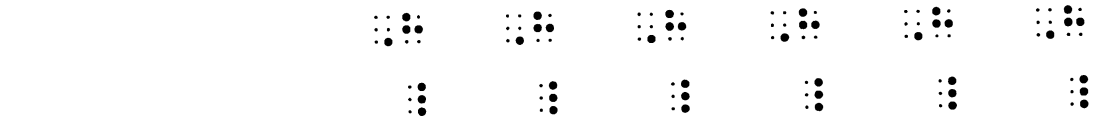
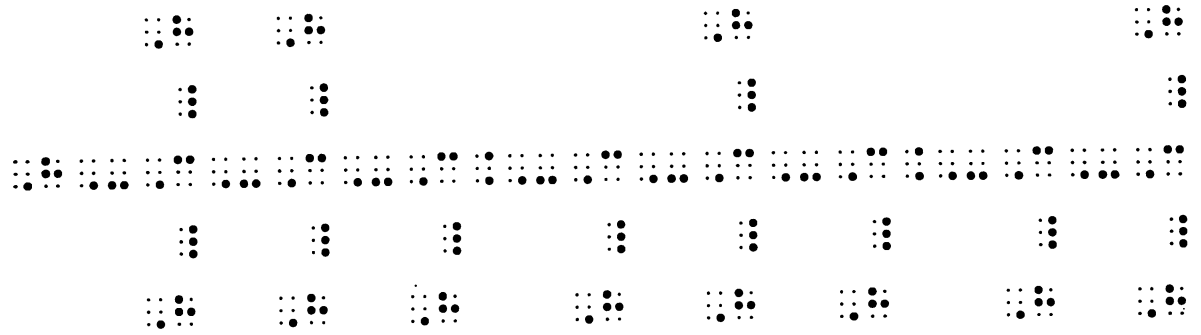
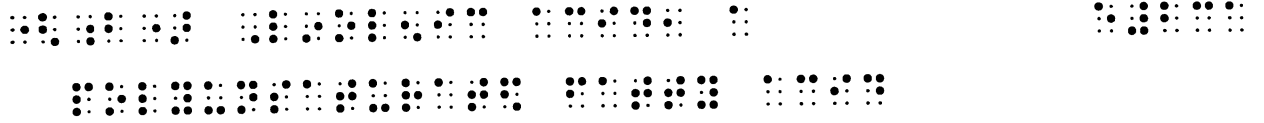
0.00244 M
0.00122 M
0.00061 M
0.000305 M

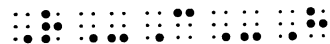
0.00122 M
0.00061 M
0.000305 M
0.000152 M

0.00061 M
0.000305 M
0.000152 M
0.000076 M

0.000305 M
0.000152 M
0.000076 M
0.000038 M

0.000192 M
0.000096 M
0.000048 M
0.000024 M





1. The first part of the experiment was to determine the molar mass of the unknown compound.

2. The second part of the experiment was to determine the empirical formula of the unknown compound.

3. The third part of the experiment was to determine the molecular formula of the unknown compound.

4. The fourth part of the experiment was to determine the percent composition of the unknown compound.

5. The fifth part of the experiment was to determine the limiting reagent in the reaction.

6. The sixth part of the experiment was to determine the theoretical yield of the reaction.

7. The seventh part of the experiment was to determine the percent yield of the reaction.

8. The eighth part of the experiment was to determine the limiting reagent in the reaction.

9. The ninth part of the experiment was to determine the theoretical yield of the reaction.

10. The tenth part of the experiment was to determine the percent yield of the reaction.

11. The eleventh part of the experiment was to determine the limiting reagent in the reaction.

12. The twelfth part of the experiment was to determine the theoretical yield of the reaction.

13. The thirteenth part of the experiment was to determine the percent yield of the reaction.

14. The fourteenth part of the experiment was to determine the limiting reagent in the reaction.

15. The fifteenth part of the experiment was to determine the theoretical yield of the reaction.

16. The sixteenth part of the experiment was to determine the percent yield of the reaction.

17. The seventeenth part of the experiment was to determine the limiting reagent in the reaction.

18. The eighteenth part of the experiment was to determine the theoretical yield of the reaction.

19. The nineteenth part of the experiment was to determine the percent yield of the reaction.

20. The twentieth part of the experiment was to determine the limiting reagent in the reaction.

21. The twenty-first part of the experiment was to determine the theoretical yield of the reaction.

22. The twenty-second part of the experiment was to determine the percent yield of the reaction.

23. The twenty-third part of the experiment was to determine the limiting reagent in the reaction.

24. The twenty-fourth part of the experiment was to determine the theoretical yield of the reaction.

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