### Section 3: Signs of Operation and Comparison

#### Operation signs:

<table>
<thead>
<tr>
<th>Braille</th>
<th>Print</th>
<th>Unicode</th>
<th>Unicode name, alternate names [description]</th>
</tr>
</thead>
<tbody>
<tr>
<td>⚅</td>
<td>+</td>
<td>002b</td>
<td>plus</td>
</tr>
<tr>
<td>⚅</td>
<td>−</td>
<td>2212</td>
<td>minus [when distinguished from hyphen]</td>
</tr>
<tr>
<td>⚅</td>
<td>×</td>
<td>00d7</td>
<td>multiplication, times [cross]</td>
</tr>
<tr>
<td>⚅</td>
<td>⋅</td>
<td>22c5</td>
<td>dot operator, times [midline dot]</td>
</tr>
<tr>
<td>⚅</td>
<td>⋅</td>
<td>00b7</td>
<td>middle dot, times [midline dot]</td>
</tr>
<tr>
<td>⚅</td>
<td>⋅</td>
<td>2219</td>
<td>bullet operator, times [midline dot]</td>
</tr>
<tr>
<td>⚅</td>
<td>÷</td>
<td>00f7</td>
<td>division, divided by [horizontal line dotted above and below]</td>
</tr>
<tr>
<td>⚅</td>
<td>±</td>
<td>00b1</td>
<td>plus-minus, plus-or-minus [plus over minus]</td>
</tr>
<tr>
<td>⚅</td>
<td>‹</td>
<td>2213</td>
<td>minus-or-plus [minus over plus]</td>
</tr>
<tr>
<td>⚅</td>
<td>:</td>
<td>2236</td>
<td>ratio, is to [colon]</td>
</tr>
<tr>
<td>⚅</td>
<td>º</td>
<td>2218</td>
<td>ring operator, composite function, hollow dot [midline hollow dot]</td>
</tr>
<tr>
<td>⚅</td>
<td>*</td>
<td>2217</td>
<td>asterisk operator, star [midline asterisk]</td>
</tr>
</tbody>
</table>

#### Comparison signs:

<table>
<thead>
<tr>
<th>Braille</th>
<th>Print</th>
<th>Unicode</th>
<th>Unicode name, alternate names [description]</th>
</tr>
</thead>
<tbody>
<tr>
<td>⚅</td>
<td>=</td>
<td>003d</td>
<td>equals</td>
</tr>
<tr>
<td>⚅</td>
<td>≠</td>
<td>2260</td>
<td>not equal to [cancellation line through an equals]</td>
</tr>
<tr>
<td>⚅</td>
<td>&lt;</td>
<td>003c</td>
<td>less-than</td>
</tr>
<tr>
<td>⚅</td>
<td>&gt;</td>
<td>003e</td>
<td>greater-than</td>
</tr>
<tr>
<td>⚅</td>
<td>≤</td>
<td>2264</td>
<td>less-than or equal to [bar under less-than]</td>
</tr>
<tr>
<td>⚅</td>
<td>≥</td>
<td>2265</td>
<td>greater-than or equal to [bar under greater-than]</td>
</tr>
<tr>
<td>⚅</td>
<td>≪</td>
<td>226a</td>
<td>much less-than [nest of two less-than signs]</td>
</tr>
<tr>
<td>⚅</td>
<td>≫</td>
<td>226b</td>
<td>much greater-than [nest of two greater-than signs]</td>
</tr>
<tr>
<td>⚅</td>
<td>≡</td>
<td>2261</td>
<td>identical to, congruent to, equivalent to [three horizontal lines]</td>
</tr>
</tbody>
</table>
3.1 Spacing

Note: The presence or absence of spaces in braille is an important aid to parsing mathematical expressions and equations. Print spacing is often simply a matter of printing style.

3.1.1 In general, signs of operation are unspaced in braille and signs of comparison are spaced in braille.

Refer to: 3.1.2 through 3.1.6 for exceptions to this general guideline.
Examples:

\[ 2y = x + 4 \]

"2y equals x plus 4"

\[ 5 - 3 \neq 3 - 5 \]

"5 minus 3 is not equal to 3 minus 5"

\[ 3 \times 5 = 5 \times 3 = 15 \]

"3 times 5 equals 5 times 3 equals 15"

\[ 200g \times 5 = 1kg \]

"200g times 5 equals 1kg"

\[ \text{Area} = bh = 5 \cdot 3 = 15 \]

"Area equals bh equals 5 times [dot] 3 equals 15"

\[ \frac{3}{15} = 3 \div 15 = 0.2 \]

"3 over 15 equals 3 divided by 15 equals 0.2"

\[ 15 \pm 0.5 \]

"15 plus or minus 0.5"
\[ x^2 - y^2 = (x \pm y)(x \mp y) \]
"x squared minus y squared equals (x plus or minus y)(x minus or plus y)"

\[ 2 : 4 = 6 : 12 \]
"the ratio 2 to 4 equals the ratio 6 to 12"

\[ x : y \]
"the ratio x to y"

\[ 1 : 2 :: x : 6 \]
"the ratio 1 is to 2 as [two colons] x is to 6"

\[ 3 \times 1 < 3 + 1 \]
"3 times 1 is less than 3 plus 1"

\[ 4^2 > 4 + 4 \]
"4 squared is greater than 4 plus 4"

Find \( \theta \) if \( 0 \leq \theta \leq \pi \)
"Find theta if 0 is less than or equal to theta is less than or equal to \( \pi \)"
If $a \leq b$, then $-a \geq -b$
"If a is less than or equal to b, then minus a is greater than or equal to minus b"

$1 \ll 1000000$
"1 is much less than 1000000"

$1 \gg 0.0000001$
"1 is much greater than 0.0000001"

$1 = 5$(mod 4)
"1 is congruent to [three horizontal lines] 5 modulo 4"

ABCD \parallel EFGH
"ABCD is similar to [three vertical lines] EFGH"

$x \sim y$
"x is equivalent to [tilde] y"

$\triangle ABC \sim \triangle DEF$
"triangle ABC is similar to [tilde] triangle DEF"

$3.9 \times 4.1 \approx 16$
"3.9 times 4.1 is approximately equal to [tilde over horizontal line] 16"
15°C \approx -9°F
"
"15 degrees C is approximately equal to [tilde over tilde] minus 9 degrees F"

\[15°C \approx -9°F\]

\[ABC \cong XYZ\]
"
"ABC is congruent to [tilde over equals sign] XYZ"

\[ABC \cong XYZ\]

\[\pi \div 3.142\]
"
"pi is approximately equal to [equals sign dotted above and below] 3.142"

\[\pi \div 3.142\]

\[x \div y \text{ implies } y \div x\]
"
"x is approximately equal to [equals sign dotted above and below] y implies y is approximately equal to [equals sign dotted above and below] x"

\[x \div y \text{ implies } y \div x\]

\[(k, l) \div (m, n)\]
"
"(k, l) is equivalent to [equals sign dotted above and below] (m, n)"

\[(k, l) \div (m, n)\]

or

\[(k, l) \div (m, n)\]

\[46 \times 32 \equiv 50 \times 30\]
"
"46 times 32 is approximately equal to [equals sign with bump in top bar] 50 times 30"

\[46 \times 32 \equiv 50 \times 30\]
\[
\frac{2}{3} \approx 0.67
\]
"two-thirds is approximately equal to [equals sign with bump in top bar] 0.67"

\[\nu_1 \approx 0\]
"\(v\) subscript 1 is approximately equal to [equals sign with bump in top bar] 0"

If \(y \propto x\) then \(y = kx\)
"If \(y\) is proportional to \(x\) then \(y\) equals \(kx\)"

\[PQ \parallel KL \text{ and } PQ \perp XY\]
"\(PQ\) is parallel to \(KL\) and \(PQ\) is perpendicular to \(XY\"

3.1.2 Signs of operation may be spaced when they are first taught, before transitioning to normal spacing practice.

Examples:

\[3 + 5 = 8\]
"3 plus 5 equals 8"

\[8 - 5 = 3\]
"8 minus 5 equals 3"
3.1.3 Signs of comparison are unspaced when they appear in an expression which is not on the base line.

Examples:

\[ \sum_{n=1}^{10} 3n \]
"the sum from n equals 1 to 10 of 3n"

\[ \int_{x=1}^{x=5} x^2 \, dx \]
"the integral from x equals 1 to x equals 5 of x squared dx"

3.1.4 Signs of comparison may be unspaced in order to avoid dividing an expression between braille lines.

Example:

\((x + 10)(x - 4) \leq 7x^2\)
"(x plus 10)(x minus 4) is less than or equal to 7 x squared"

3.1.5 Signs of operation may be spaced to enhance the parsing of an expression which includes spaces between quantities and their units.

Refer to: 9.3 for spacing of signs of operation with function names and their arguments.

Examples:

2 ft + 4 ft = 6 ft
"2 ft plus 4 ft equals 6 ft"

57.2 mm ÷ 10 = .572 cm
"57.2 mm divided by 10 equals .572 cm"
3.1.6 Follow print spacing for signs of operation and comparison when the adjacent text is not a wholly mathematical expression.

*Examples:*

- x axis from −4 to +5
  
  "x axis from negative 4 to positive 5"
  
  \[ \text{x axis from } -4 \text{ to } +5 \]

  The answer is \(-\frac{1}{2}\).

- "The answer is negative one half."
  
  \[ \text{The answer is } -\frac{1}{2}. \]

- The area receives <2 inches of rain.
  
  "The area receives less than 2 inches of rain."
  
  \[ \text{The area receives } <2 \text{ inches of rain.} \]

- a range of \(\approx 4\mu g\)
  
  "a range of approximately [equals sign with bump in top bar] 4 micrograms"
  
  \[ \text{a range } \approx 4\mu g \]

3.2 **Minus**

*Note:* The print minus is very similar to the print hyphen but is normally shown as a slightly longer horizontal line, particularly in technical material.

3.2.1 It is permissible to use a braille hyphen for a minus which is indistinguishable from a hyphen in print.

*Example:*

- current balance - initial balance = accumulated interest
  
  "current balance minus [hyphen] initial balance equals accumulated interest"
  
  \[ \text{current balance - initial balance } = \text{ accumulated interest} \]
3.3 **Positive and negative numbers**

3.3.1 Use a superscript indicator when a plus or minus, indicating a positive or negative number, is in the superscript position.

*Note:* It is permissible to describe the superscript position in a transcriber’s note instead of using a superscript indicator.

*Examples:*

Evaluate $-2 + -3$

"Evaluate negative 2 plus negative 3 [with the minus signs in the superscript position in print]"

$\text{Evaluate } 2 \mbox{ } 3^+ + 2 \mbox{ } 3^-$

$-2^\circ \text{C or } +2^\circ \text{F}$

"minus 2 degrees C or plus 2 degrees F [with the minus and plus signs in the superscript position in print]"

[open tn]Print uses a superscript plus for a positive number and a superscript minus for a negative number. The superscript position is not shown in braille.[close tn]

$\text{Print uses a superscript plus } + \text{ a positive number } +$

$\text{a superscript minus } - \text{ a negative number} \pm$

$\text{superscript position is not shown in braille}$

$-2^\circ \text{C or } +2^\circ \text{F}$

3.4 **Hollow dot**

3.4.1 Use the hollow dot to represent the mathematical sign of operation.

*Refer to:* *Rules of Unified English Braille*, section 3.5 for the bullet and section 3.11 for the degree sign.
Examples:

\[(f + g) \circ h = f \circ h + g \circ h\]

"(f plus g) hollow dot h equals f hollow dot h plus g hollow dot h"

\[(f \circ g)(x) = f(g(x))\]

"(f hollow dot g)(x) equals f(g(x))"

3.5 Asterisk, star

3.5.1 Use the braille asterisk to represent the print midline asterisk used as a sign of operation in mathematics. It is generally brailled unspaced.

Note: Both the midline asterisk and the raised asterisk are represented by the same braille sign.

Refer to: Section 17, Computer Notation for the asterisk as an ASCII symbol; and Rules of Unified English Braille, section 3.3 for the asterisk used in literary material.

Examples:

\[3 \ast 2 = 2 \ast 3\]

"3 times [asterisk] 2 equals 2 times [asterisk] 3"

\[\ast\text{ is distributive over } \circ \text{ if } a \ast (b \circ c) = (a \ast b) \circ (a \ast c)\]

"asterisk is distributive over hollow dot if a asterisk (b hollow dot c) equals (a asterisk b) hollow dot (a asterisk c)"